



# Reserve Bank of New Zealand Analytical Notes

## Market perceptions of exchange rate risk AN 2012/12

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## NON-TECHNICAL SUMMARY<sup>1</sup>

From data on prices for foreign exchange options, which can be broadly thought of as insurance products, we can extract implied market perceptions of future exchange rate risk. Intuitively, if markets put much more weight on the probability of depreciation than of appreciation, the price of the respective options will reflect that difference. We can calculate the expected probability that the exchange rate will be above (or below) a given level in the future and see how these probabilities change over time.

Our analysis suggests that the behaviour of the NZD/USD market has changed significantly since the Global Financial Crisis. The onset of the Global Financial Crisis saw the probability densities become increasingly dispersed as market uncertainty grew. The balance of risks for the NZD/USD also changed significantly with the market moving to much more weight on the chance of depreciation than of appreciation. Indeed, despite the current low volatility environment for the NZD/USD, the market continues to price significant risk of depreciation over appreciation. These changes in the characteristic of the distributions indicate market participants have changed the way they price risk.

Statistical tests suggest the densities calculated are, broadly speaking, well-calibrated. Data of this sort provide a useful insight for the Reserve Bank in its monitoring of foreign exchange markets and its assessment of changes in risk around the New Zealand dollar market.

## INTRODUCTION

Derivative prices are a rich source of information about expected future returns. The prices at which market participants are willing to transact provides powerful insights into expectations. Forwards and futures markets can reflect the market's *average* expectation of the future, while *options* markets can reflect the *distribution* of implicit expectations.

In this paper we use over-the-counter option prices to look at implicit market expectations of future exchange rates. We derive probability density functions for the NZD/USD based on options of up to 12-months duration. The distributions show the dispersion and skewness of market expectations, how much probability the market attaches to large moves, and how these expectations change over time.

The probability densities are assessed statistically using probability integral transformations (PITs). PITs help to judge whether densities are biased and whether the widths of the densities are well-calibrated.

## OPTIONS MARKET

### What is an option?

An option<sup>2</sup> is a contract that gives the right, but not the obligation, to buy or sell an asset, at a set date in the future, at a set price called the strike price. In its most basic sense, if there is a positive probability that the underlying asset will be worth *more* than the strike price in the future, then the option to *buy* the asset is valuable (buy a call option). Conversely, if there is a positive probability that the underlying asset will be worth *less* than the strike price in the future, then the option to *sell* the asset is valuable (buy a put option). An investor will decide to not exercise the option when it is not profitable. There is a premium for taking out the option contract, so there is a fixed difference between payoff and profitability for an option.

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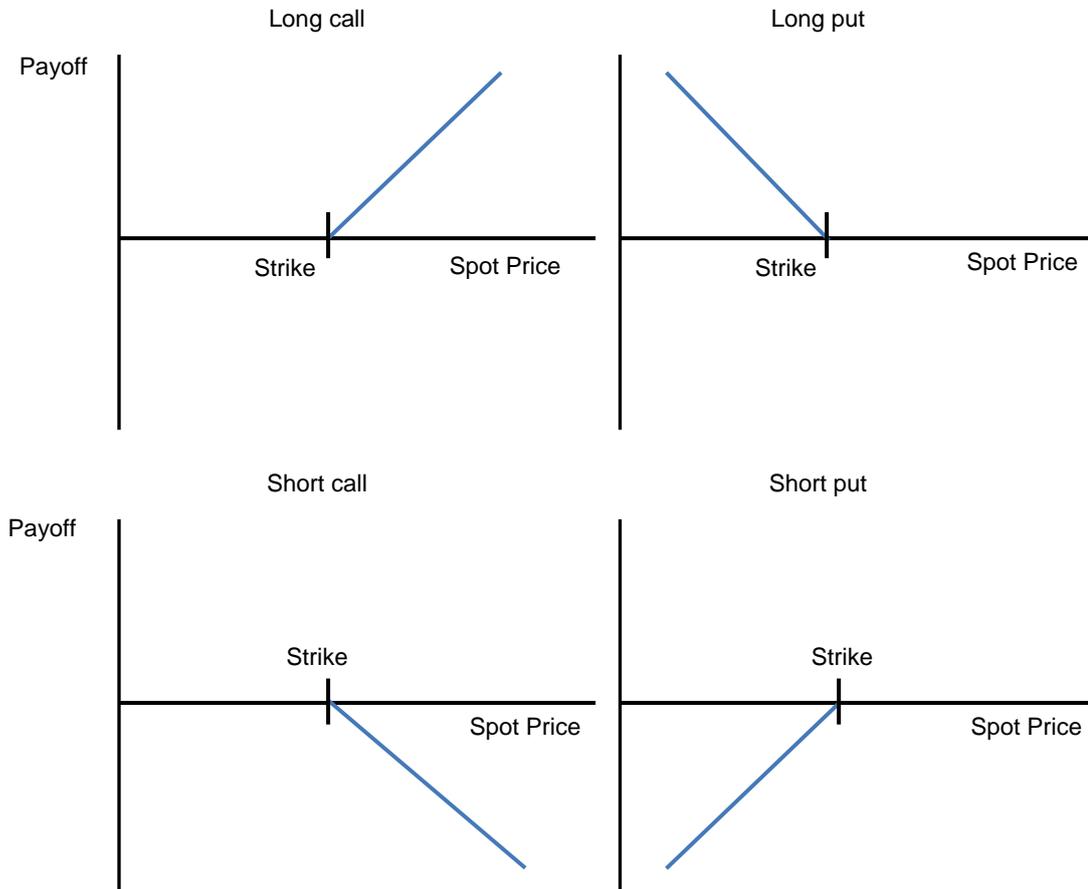
<sup>1</sup> I would like to thank Michael Reddell, Lauren Rosborough, Christie Smith, and Jason Wong for helpful comments, as well as Andrew Blake (Bank of England) and Ole Rummel (Bank of England) for our discussions.

<sup>2</sup> There are two types of common options: European and American. An American option can be exercised at any time, up to and including the expiry date, whereas a European option cannot be exercised early. We use European options in this paper.

Payoffs from four option positions are shown in figure 1 (these charts abstract from the fixed premium). Options are a zero sum game, payoffs equal zero across the two parties. The payoff from the long call is -1 times the payoff from a short call. A short put has -1 times the payoff from a long put.

Figure 1

## Option positions



Source: RBNZ

## NZD/USD options market liquidity

Probability distributions derived from options will only accurately describe changing market expectations if the contracts are actively traded. Consequently, market liquidity is important for the analysis presented in this paper. The latest Bank for International Settlements (BIS) survey on currency market liquidity finds that options account for 4.4 percent of New Zealand's exchange market turnover (BIS (2010)), a little less than the global average of 5.2 percent. The same survey places the New Zealand dollar as the 10<sup>th</sup> most traded currency, accounting for 1.6 percent of global foreign exchange market turnover. Although the New Zealand options market is small it appears to be reasonably liquid, where 4.4 percent of turnover is equivalent to an average of approximately USD1.4bn a day<sup>3</sup>. Therefore we can expect New Zealand exchange rate options to be reasonably accurate at summarising market expectations. An inspection of daily data supports this conclusion: the data can be noisy but there do appear to be meaningful patterns. Graphs of the options data are presented in *Appendix A*.

<sup>3</sup> We note that liquidity on shorter-dated options (less than 6 months) is much better than longer-dated options.

## THE MODEL

### How can we extract probabilities from option prices?

Any given asset can have a number of option contracts available. The put and call options described above are typically referred to as 'vanilla options'. Option strategies can include combinations of vanilla puts and calls. For a particular asset, we can gather various contracts that have different option prices. These prices tell us about how the market sees the chances of the asset being above or below the strike price in the future.

In this note, we use three option strategies that are used by market participants: straddle, risk reversal, and strangle (further detail is provided in *Appendix A*).<sup>4</sup> These strategies are combinations of 'vanilla' puts and calls described above. Market prices for these strategies provide valuable insight on the distribution of possible outcomes. A straddle gives the expected variance (volatility of the asset's price), a risk reversal gives the expected skewness (direction of the asset's price relative an average expectation), and a strangle gives the expected kurtosis (chance of large moves of the asset's price). Taking these prices relative to one another we can calculate a distribution.

The model used in this paper was developed by Malz (1997) and has been previously used at the Reserve Bank in Gereben (2002). Further detail of the method is provided in *Appendix B* but the key assumptions are as follows: we do not assume a particular distribution for possible outcomes; we use the Black-Scholes pricing formula to convert option prices but this does not mean we adopt Black-Scholes assumptions<sup>5</sup>; and we assume risk-neutrality.

The assumption of risk neutrality means that the probability density is based on the set of probabilities that investors would attach to future outcomes if they care only about *expected* gains or losses. However, if investors are risk averse and would readily pay a premium to protect against excessive volatility then that risk premia will drive a wedge between the probabilities inferred using the Malz method and the true probabilities. If this premia is fairly constant then there is little consequence for the distribution in relative terms. Also, it can be argued that while investors that take out options could be risk averse, the other side of the transaction may not be. This could result in the degree of risk aversion being less than is often thought. In future work we intend to allow for risk aversion. This will allow an assessment of the degree of risk aversion present and how it changes over time. This alternative model will then be tested and compared against the benchmark Malz method.

This class of model is used at other central banks, such as the Federal Reserve, Bank of England, and the European Central Bank.<sup>6</sup> The Federal Reserve Bank of Minneapolis and the Bank of England publish risk-neutral probability densities on their websites for various assets.<sup>7</sup>

### Key points for interpreting probability density functions:

Probability density functions have a number of features we can make use of, particularly in the sense of how these features change over time. Four key features of any probability density are highlighted below.

- 1) **Central tendency:** has the average value moved? The mode is the peak of the density.
- 2) **Volatility:** is the distribution tightly concentrated around the mode or is it spread out? When the distribution spreads out, meaning there is more variance, it indicates that investors are more uncertain about the future. In the graph below, the grey distribution has lower volatility than the black distribution.

<sup>4</sup> Note that we are using data from 2005 because of data limitations.

<sup>5</sup> Black-Scholes is a famous option pricing model, but it has many empirical short-comings.

<sup>6</sup> See, for example, Bliss and Panigirtzoglou (2004), Clews, Panigirtzoglou and Proudman (2000), Vincent-Humphreys and Gutierrez (2010)

<sup>7</sup> Bank of England option implied probability density:

<http://www.bankofengland.co.uk/statistics/Pages/impliedpdfs/default.aspx>

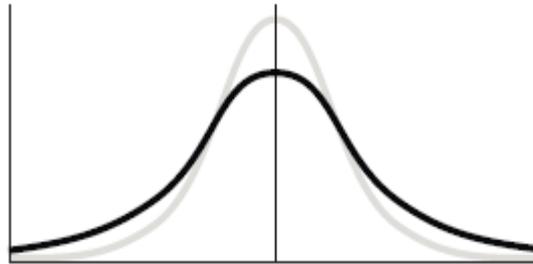
The Federal Reserve Bank of Minneapolis option implied probability density:

<http://www.minneapolisfed.org/banking/assetvalues/>

Figure 2

## Volatility

Probability



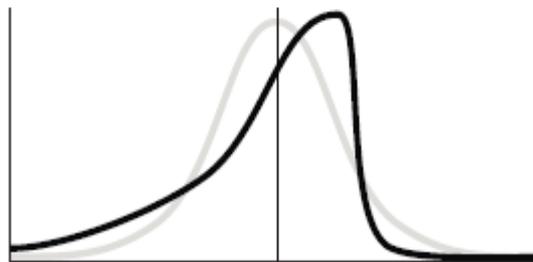
Possible outcomes

- 3) **Skewness:** is the distribution centred on the mode or is it concentrated in a particular direction? When the distribution is skewed, it indicates that investors see more risk for the asset to move in a particular direction. In the graph below, the grey distribution is symmetric and the black distribution is negatively skewed. The negative skew indicates that large price falls are seen as being more likely than large price increases.

Figure 3

## Skewness

Probability



Possible outcomes

- 4) **Tails or kurtosis:** does the distribution have more or less mass in the tail of the distribution? The more mass that is in the tail, the larger the probability that the market attaches to large moves in the asset's price. In the graph below, the black distribution shows fatter tails than the grey distribution, meaning extreme outcomes are seen as being more likely. A distribution with kurtosis greater than 3 is commonly described as having 'fat tails' relative to a normal distribution.

Figure 4

## Kurtosis

Probability



Possible outcomes

### NZD/USD implied probability density functions

Using the method outlined above, we create probability densities for the NZD/USD using daily 1, 2, 3, 6, 9, and 12 month option prices.

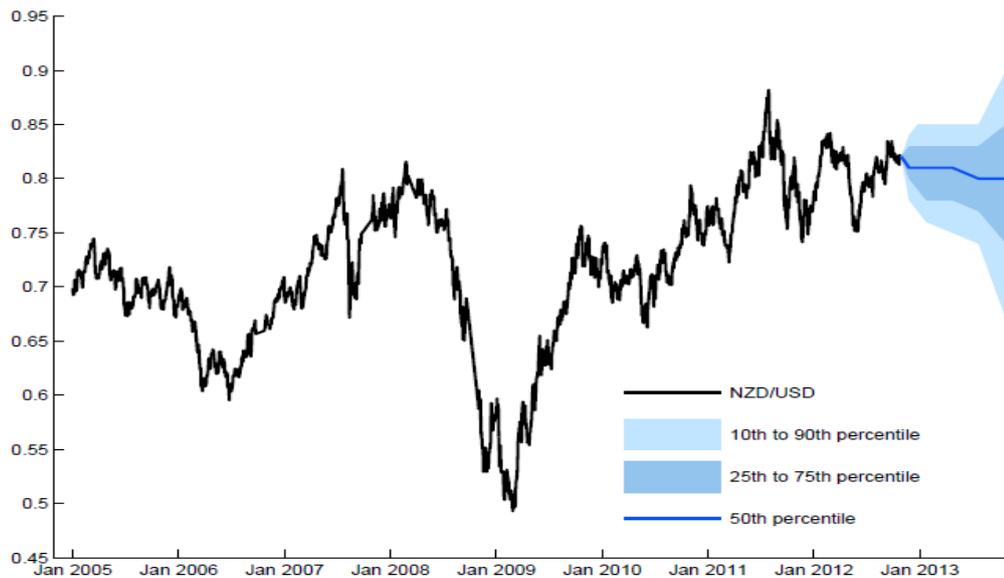
#### *Market perceptions of exchange rate risk as of 30 October 2012*

For any particular trading day, we can create a fan chart using NZD/USD option contracts for different horizons (figure 5). In the example below we look at 30 October 2012. The fan chart is akin to a sequence of density forecasts. The layers in the fan chart represent percentiles. The lightest blue area shows 80 percent probability of the exchange rate being in that band at each date in the future.

From the fan chart in figure 5 we can see that the market is pricing in a gradual depreciation of the NZD/USD to 0.80 from the spot price of 0.822 (dark blue line, the 50<sup>th</sup> percentile). The bands show that there is a significant degree of uncertainty, and that the probability mass is skewed towards depreciation at each horizon (see table 1 for summary statistics). The uncertainty around the future path of the exchange rate increases further into the future, with the 12-month contract showing the largest degree of uncertainty. The distributions for longer-dated options are influenced by both increased market uncertainty and diminished market liquidity.

Figure 5

#### NZD/USD fan chart, up to 12 months ahead



Source: RBNZ calculations, Reuters, Bloomberg

Table 1: Summary statistics<sup>8</sup>

	1 month	2 months	3 months	6 months	9 months	12 months
Mean	0.81	0.81	0.81	0.81	0.80	0.80
Standard deviation	0.022	0.031	0.039	0.041	0.043	0.09
Skewness	-0.18	-0.30	-0.38	-0.38	-0.33	-0.39
Kurtosis	3.17	3.2	3.22	3.19	3.14	3.01
Probability of:						
10% depreciation	0.0%	0.7%	2.8%	4.4%	4%	19%
10% appreciation	0.0%	0.15%	1.2%	1.2%	2.3%	16%

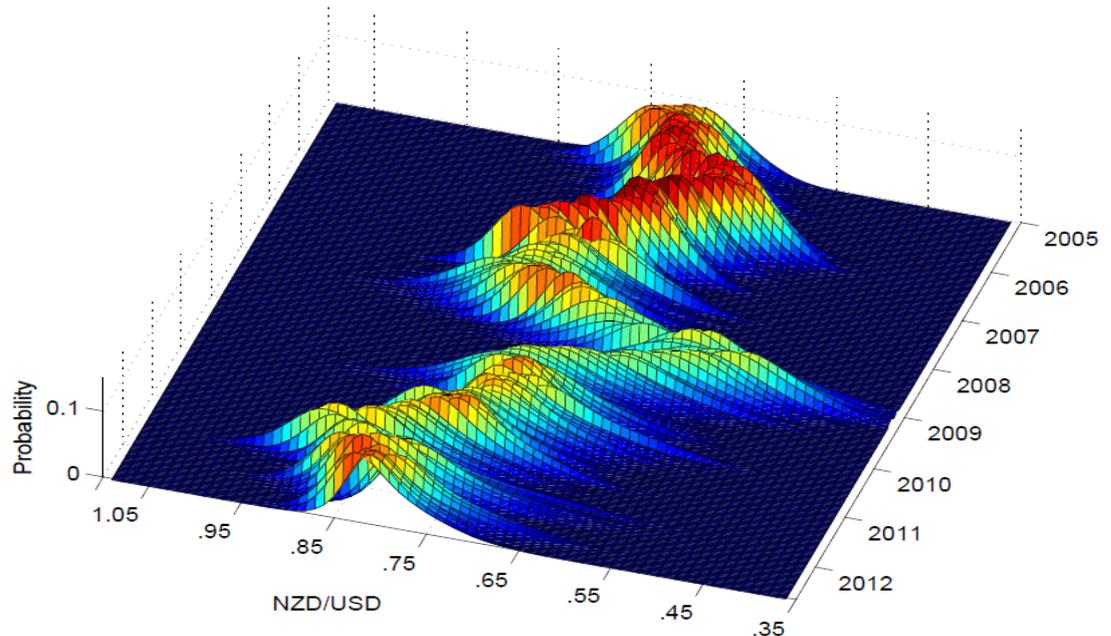
<sup>8</sup> A negative skew means large depreciation of the NZD/USD is more likely than large appreciation. Kurtosis greater than three indicates 'fat tails' relative to a normal distribution i.e. there is more probability of extreme events.

*How have the distributions changed overtime?*

For each trading day we can create a density and see how these change over time. Figure 6 shows 3-month probability densities based on the NZD/USD between 2005 and 2012. The densities are colour coded, akin to a contour map of hills or a heat map. The bottom of the density with the least amount of probability mass is a dark blue. The area with the most amount of probability mass is red. The areas warm from blue to red as the probability mass increases.

Figure 6

NZD/USD three-month distributions overtime



Source: RBNZ calculations, Reuters, Bloomberg

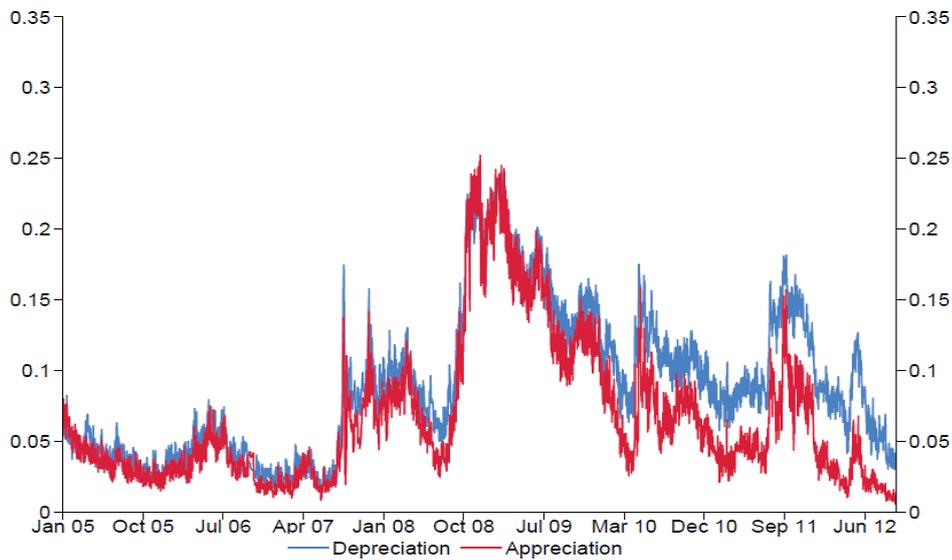
The onset of the Global Financial Crisis (GFC) in the second half of 2007 saw the densities become more dispersed as market uncertainty grew (shown by the smaller red surface area). The densities also became increasingly skewed with the market putting much more weight on depreciation rather than appreciation (shown by the longer tails on the right hand side of the distributions). Since early 2011, market pricing has become less dispersed but the persistent skew remains. Although volatility is slowly returning to 'normal' the market is now more wary of the NZD/USD depreciating. Indeed, the distributions now show persistently 'fatter tails', meaning that the market now assigns more probability to large moves in the NZD/USD (further detail is provided below). These changes in the characteristics of the distributions indicate market participants have changed the way they price risk and position themselves in the NZD/USD market.<sup>9</sup>

An intuitive way of looking at how the market prices risk is to explicitly look at the probability of a large move in the NZD/USD. Figure 7 shows the probability of a 10 percent appreciation and 10 percent depreciation. Towards the end of 2008 (the height of the GFC) the market was incredibly uncertain about the future. The probability of a 10 percent move in either direction, over a three month horizon, was around 25 percent. The probability of such a large move is much less now, at around 3 percent for depreciation and 1 percent for appreciation. However, a persistent gap between the two probabilities has opened up since 2009. For more insight into this change, we look at the moments of the distribution in figures 8, 9, and 10.

<sup>9</sup> Discussions with option traders indicate that the NZD/USD had negative risk reversals throughout 2000-2005. However, given the data limitations we have not been able to quantify the skew.

Figure 7

## NZD/USD three-month probabilities of 10 percent appreciation and depreciation

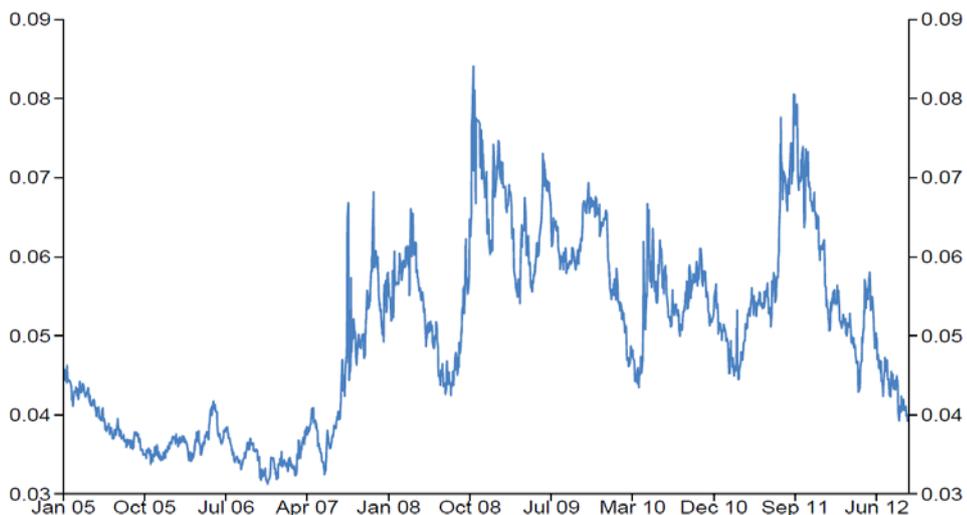


Source: RBNZ calculations, Reuters, Bloomberg

The onset of the GFC saw a stark change in how the options market perceives the balance of risks for the exchange rate. The changes are more pronounced when looking at the higher moments of the distributions (skewness and kurtosis). The expected volatility of the NZD/USD has been returning to historical levels or 'normalising' (figure 8). Realised volatility of the NZD/USD has also been trending lower.

Figure 8

## Standards deviation of three-month densities

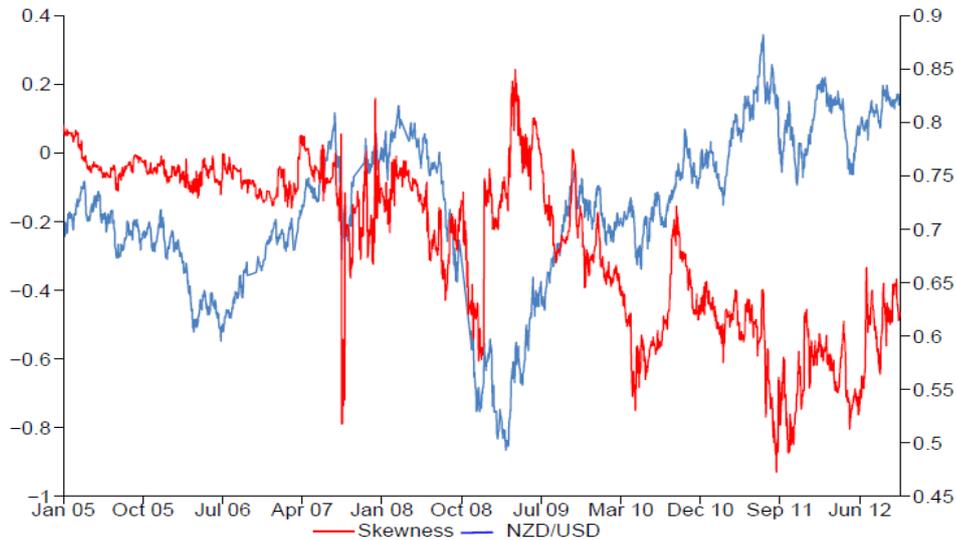


Source: RBNZ calculations, Reuters, Bloomberg

Since mid-2007 the market has put much more weight on the risk that the NZD/USD could depreciate (figure 9). This could reflect the market repricing the risk of a sharp depreciation, but it could be related to the level of the NZD/USD (the market pricing in a degree of mean reversion).

Figure 9

## Skewness of three-month densities

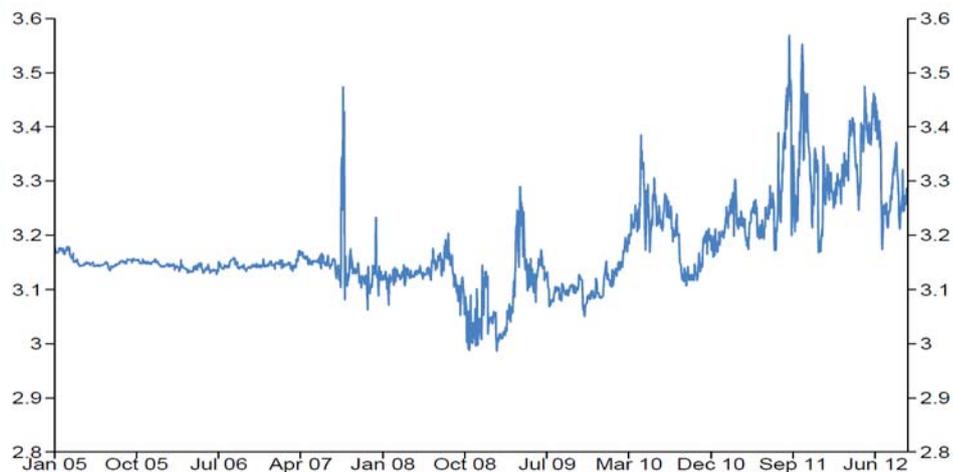


Source: RBNZ calculations, Reuters, Bloomberg

The behaviour of the tails of the distributions has also changed substantially since mid-2007 (figure 10).<sup>10</sup> Leading into the GFC, the size of the tails was fairly stable. At the height of the GFC, the 3-month distribution became increasingly dispersed (figure 8). This meant the tails of the distributions thinned because market uncertainty was so great that large changes were given significant weight, resulting in the whole distribution flattening (refer to figure 6). Over time the NZD/USD distributions have become less dispersed but the tails of the distributions are now much fatter than pre-GFC.<sup>11</sup> This means the market is now much more wary of extreme (especially downward) moves in the NZD/USD. That gap that has opened between the probabilities of a 10 percent appreciation and depreciation is consistent with this.

Figure 10

## Kurtosis of three-month densities



Source: RBNZ calculations, Reuters, Bloomberg

<sup>10</sup> Note that kurtosis greater than 3 indicates the distribution has fat tails relative to a normal distribution.

<sup>11</sup> Note that we are comparing how the second and fourth moments are behaving over time. The standard deviation is the second moment, skewness is the third moment, and kurtosis is the fourth moment.

This note focuses on the NZD/USD, however the AUD and NZD often trade in similar patterns against the USD. Our preliminary analysis on option prices for the AUD/USD shows very similar patterns found for the NZD/USD. In particular, we find that the AUD/USD distributions are typically skewed towards depreciation. At the height of the GFC, the skew became more pronounced and this feature has persisted.

### DEGREE OF CONFIDENCE IN THE METHOD: USING PITS

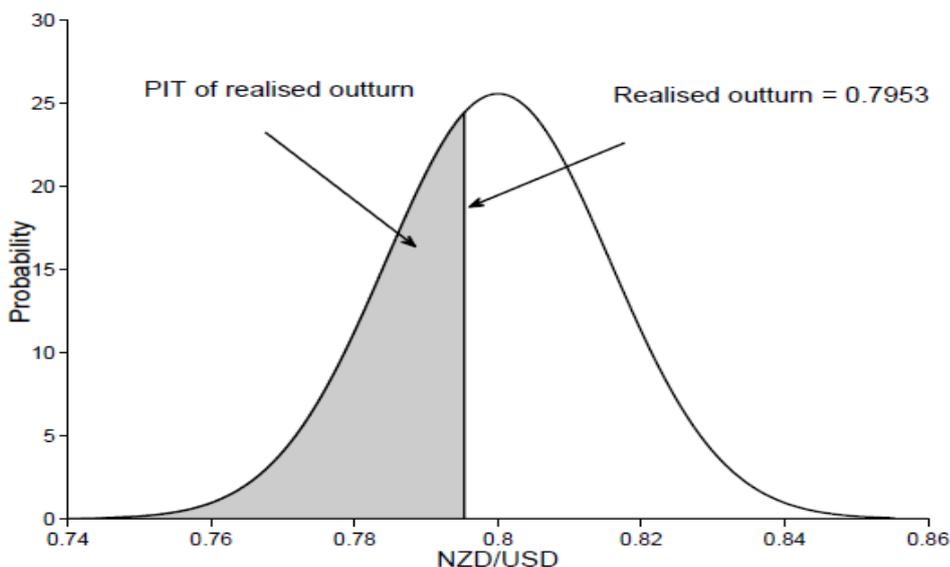
#### How good are the distributions?

The density forecasts can be assessed using probability integral transformations (PITs) for realised exchange rate values. If the method used to generate the densities is correct then the PITs will have a uniform distribution. PITs show the number of observations in each 10 percent interval of the densities: a well-specified density will have an even number of observations in each interval. Uniform PITs mean the forecast densities are unbiased and 'well-calibrated'.

Figure 11 illustrates how PITs are calculated at each point in time. The PIT is the area under the forecast probability density evaluated to the left of the realised outturn (the grey area). An in-depth explanation of PITs can be found in Bjornland et al (2010), but the method is based on the result that random variables from any continuous distribution can be converted into random variables having a uniform distribution. If the probability densities that we calculated are the 'true' distributions then we will be able to convert the forecast densities, at the realised outturns, into uniform distributions.

Figure 11

#### Example of evaluating a density



Source: RBNZ

Figure 12 shows PITs for the densities derived from option contracts of different durations. Well-calibrated distributions would have vertical bars evenly centred on the dashed horizontal line. We find that for longer duration options, too much weight is put on the chance of depreciation, relative to a well-calibrated distribution. This is less of an issue for options up to 3-month durations. Conversely, 1-month options (blue bars) tend to put slightly too much weight on appreciation.<sup>12</sup>

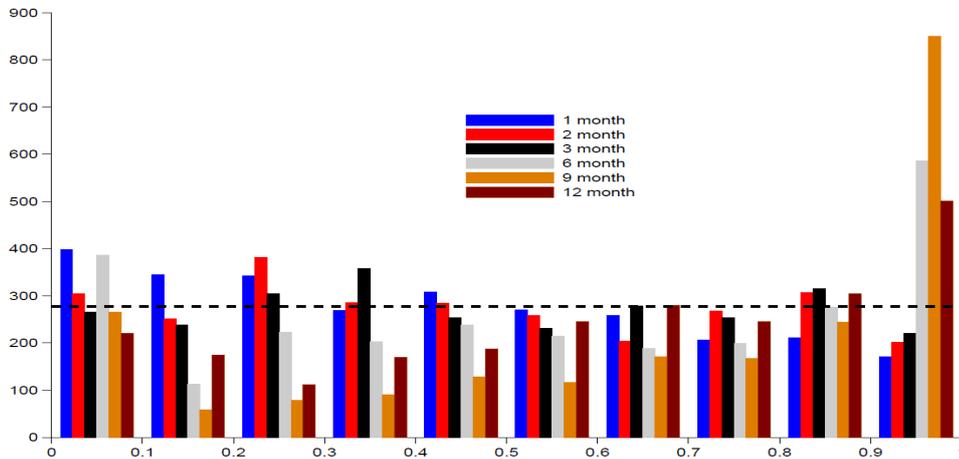
We can see that the forecast densities are broadly well calibrated. That is, their width is about right and there is broadly the right amount of PITs in the tails and centre of the distribution. If the densities were too narrow then PITs would fall in the tails of the densities more often than was warranted. If the

<sup>12</sup> Bars above the dashed line put too much weight on that probability interval and bars below the line put too little weight on the interval.

densities were too broad then realised outturns would be concentrated in the centre of the distribution and not enough in the tails.

Figure 12

PITs



Source: RBNZ calculations

## CONCLUSION

Using *option prices* we can extract estimates of the market's perception of future exchange rate risk. Specifically, we compute probability density forecasts for the NZD/USD. We find that these densities are broadly well-calibrated.

We find evidence that the behaviour of the NZD/USD has changed significantly since the GFC. Indeed, although actual volatility in the NZD/USD is quite low at present, the market continues to price significantly greater risk of depreciation than of an appreciation.

## APPENDIX A: OPTION PRICE TERMINOLOGY AND DATA

**Terminology:**

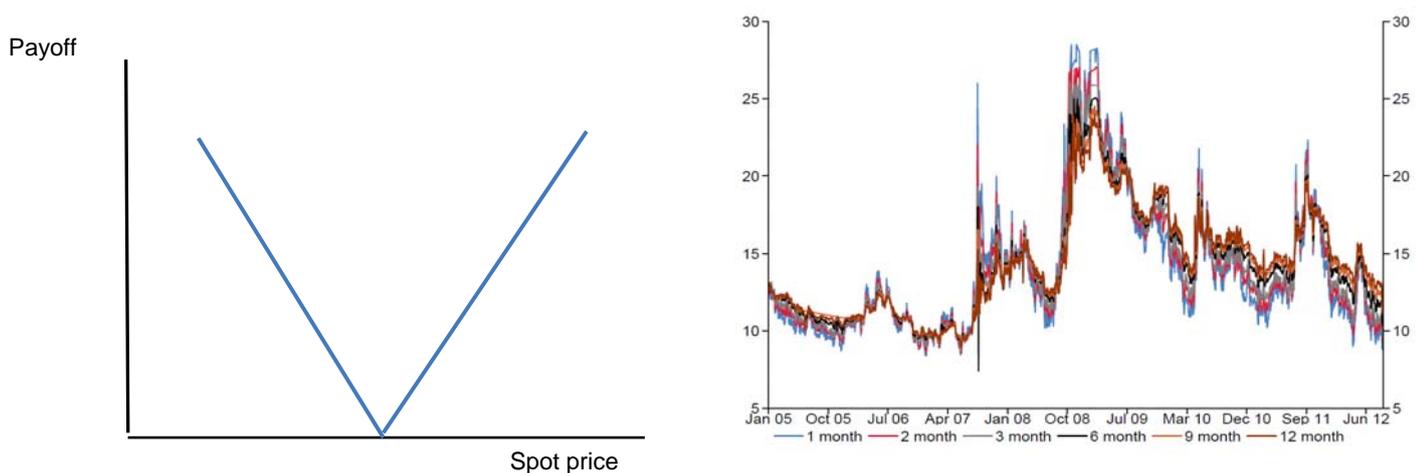
- European call option: the buyer of a call option has the right, but not the obligation, to *buy* the underlying asset at a predetermined price (strike price) at a specified date in the future. The buyer of the call option wants the price of the underlying asset to increase.
- European put option: the buyer of a put option has the right, but not the obligation, to *sell* the underlying asset at a predetermined price (strike price) at a specified date in the future. The buyer of a put option wants the price of the underlying asset to fall.
- Strike price: the predetermined price at which the asset can be bought or sold for in the future.
- “At the money” is when the strike price of the option is the same as the current spot price of the underlying asset.
- “Out of the money”: the option has no monetary value at that point in time. However, an investor may purchase an out of the money option with the view that it could become profitable. For example, an investor could buy a put option if they believe the NZD/USD will depreciate. The further out of the money an option is the less sensitive it is to changes in the price of the underlying asset.
- Delta: delta measures how an option price will move given a small change in the underlying asset. For example, an option with a delta of 0.5 moves half a cent for each 1 cent move in the NZD/USD.

**Option strategies used:***Straddle: expected variance*

A straddle is the combination of a call and a put option with the same strike price and expiry date. The pay-off from the combination is shown below (excluding premium). The pay-off increases the further the spot price is from the strike price, regardless of the direction. So the more variance that is expected, the higher the payoff expected from taking out the contract, and hence the higher price of the contract.

Figure 13

## Straddle strategy (LHS) and price of strategy (RHS)



Source: Bloomberg, RBNZ

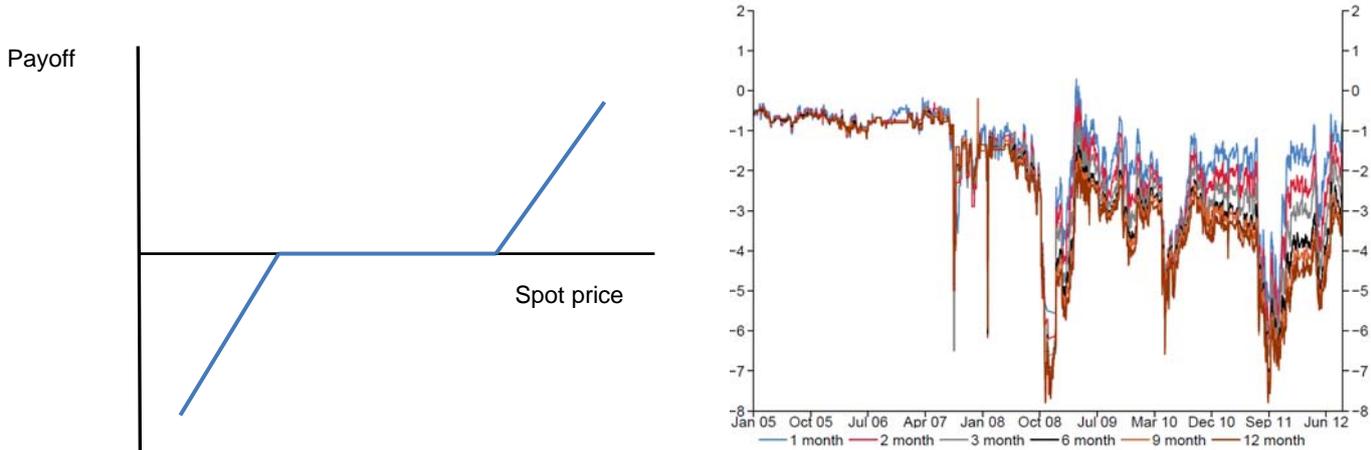
*Risk reversal: expected skewness*

A risk reversal is a combination of selling an out-of-the-money put option and buying an out-of-the-money call option, which have the same maturity. The price of a risk reversal tells us how market participants view the likely direction of the exchange rate in the future. If an appreciation is believed to be more likely than depreciation, then the call option is more likely to be profitable than a put option. In this case the price of the call option would be higher than the put option, reflecting the relative perceived profitability, and the price of the risk reversal would be positive (call price – put price). Risk reversals give the implied direction but not magnitude of an expected change in the exchange rate. We use 25-delta risk reversals, which are out-of-the-

money options where the option prices moves 0.25 cents for each 1 cent move (a standard market convention).

Figure 14

Risk reversal strategy (LHS) and price of strategy (RHS)



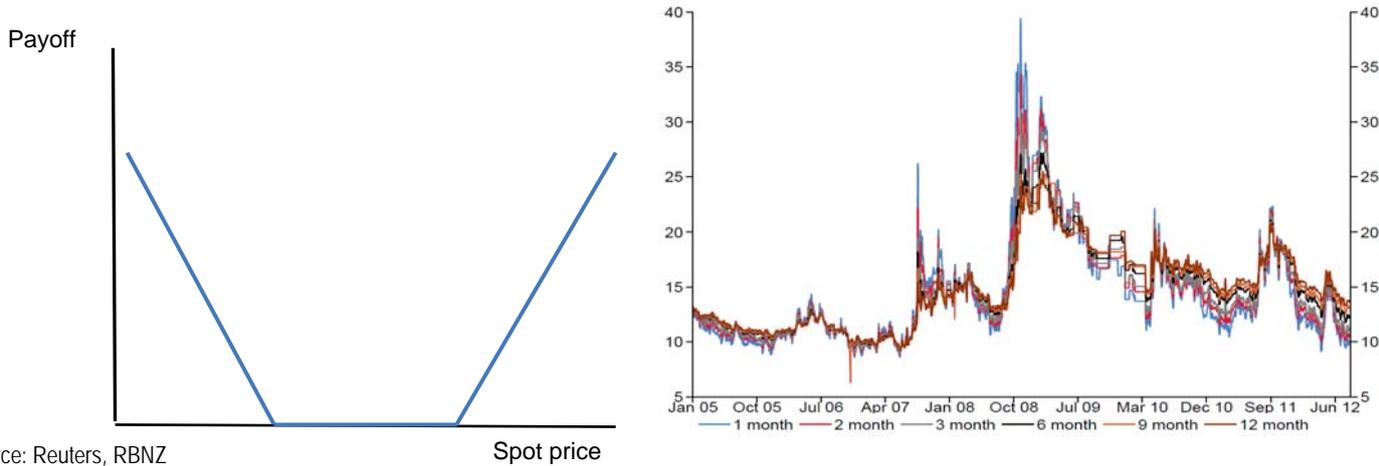
Source: Bloomberg, RBNZ

*Strangle: expected kurtosis*

A strangle is a combination of buying an out of the money put and an out of the money call option. A strangle becomes profitable at extreme levels of the exchange rate, so investors are implicitly betting on tail events. As large moves in the exchange rate are seen to become more likely, the payoff from taking out a strangle option increases and hence the price increases. Strangles will give the implied direction of the distribution's tails (fat or thin) but not the magnitude (how fat or thin). We use 25-delta strangles.

Figure 15

Strangle strategy (LHS) and price of strategy (RHS)



Source: Reuters, RBNZ

## APPENDIX B: METHODOLOGY

### The Malz Method: a smoothed volatility smile

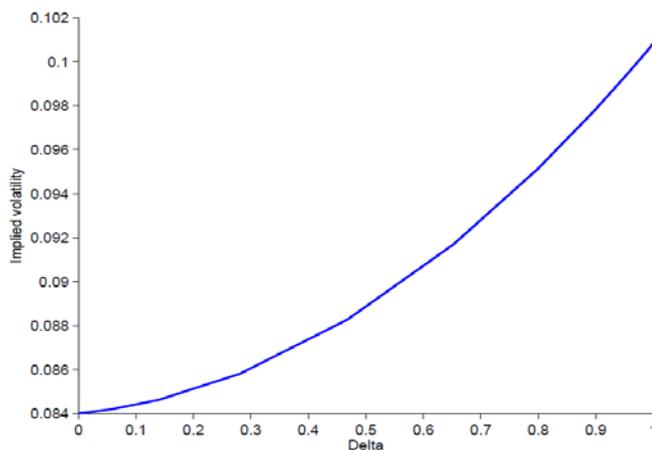
We outline the three main steps in the Malz method below. See Malz (1997) for technical detail of the method.

#### *Step 1: estimate a smoothed 'volatility smile'*

A volatility smile is estimated using the three option strategies, a risk-free interest rate, the strike price relative to the forward rate, and the time discount factor. A volatility smile describes how options on the same underlying asset with different strike prices/deltas have different implied volatilities. This is an empirical feature in options and violates an assumption made in Black-Scholes (1973). A polynomial is used to fit the curve. Other techniques involve fitting several polynomials, called a spline.

Figure 16

Volatility smile



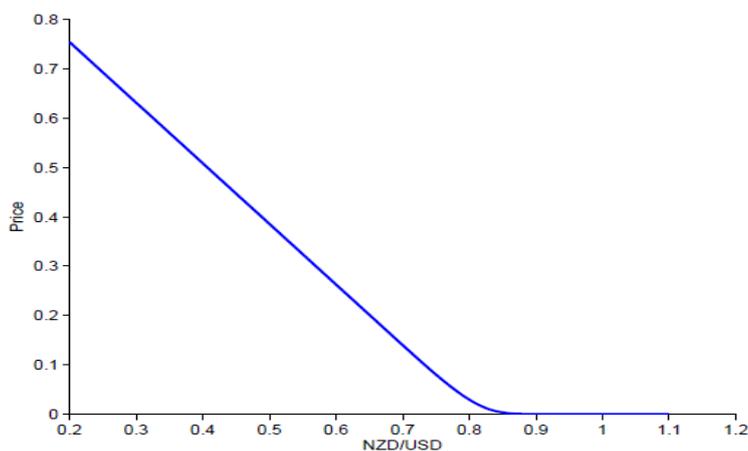
Source: Bloomberg, Reuters, RBNZ calculations

#### *Step 2: Use the volatility smile to calculate a call function*

The volatility smile is converted into a call function using the Black-Scholes pricing formula. The call function maps the call price of the option with the underlying asset's price. Crucially, this is a transformation that allows us to go from delta space to price space, but we do not adopt Black-Scholes assumptions.

Figure 17

Call price function



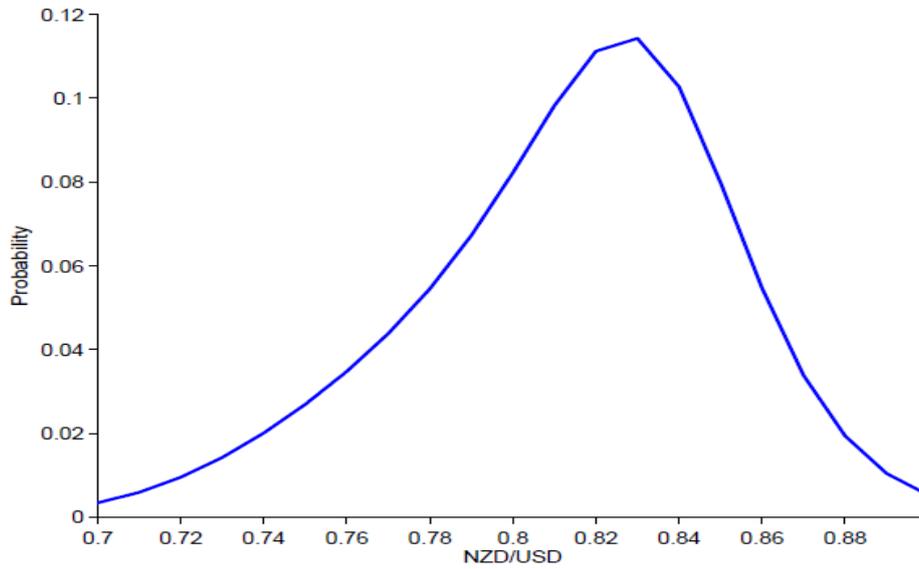
Source: Bloomberg, Reuters, RBNZ calculations

*Step 3: From the call function calculate the probability density function*

Using the result from Breeden and Litzenberger (1978), twice differentiating the call price function, we compute the risk-neutral probability density function.

Figure 18

Risk-neutral probability density



Source: Bloomberg, Reuters, RBNZ calculations

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