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MONETARY POLICY MAKING IN THE PRESENCE OF KNIGHTIAN UNCERTAINTY

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Abstract

This paper explores the extent to which Knightian uncertainty can explain why interest rates generally move in a sequence of steps in a given direction, or remain constant for some time, rather than experiencing the frequent policy reversals that commonly arise from optimal policy simulations. We categorise the types of uncertainty that have been explored to date in terms of the decision-making behaviour they imply. We conclude by suggesting that one formalisation of Knightian uncertainty is more intuitively appealing, in the context of monetary policy, than formalisations that have been previously used. Within a very simple optimal control problem, we show that our preferred formalisation can deliver interest rate paths with periods of no change, and with some modifications, can generate paths of interest rates strikingly similar to those observed in practice.

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1. Introduction

One of the stylised facts of monetary policy is that interest rates generally move in a succession of small steps in the same direction and can remain fixed at the same level for some time. However, models of optimal monetary policy behaviour tend to generate much more volatile paths of interest rates, in which policy reversals are frequent. Clearly, central bankers are not behaving the way the models predict they should. This indicates that there is a need to reconcile model generated and observed behaviour. Recently, there has been a growing body of literature that has focussed on the role of uncertainty and its effects on monetary policy implementation to this end.

One line of this research has shown that some of the volatility in interest rate paths generated by models can be eliminated by considering uncertainty induced arising from the fact that the model's parameters are non known and therefore have to be estimated. In this case, the distribution of possible outcomes is defined by the probability distribution of the parameter estimates. Because there is assumed to be a single probability distribution, the standard assumptions underlying expected utility theory apply, and the methods of solving these decision problems are essentially the same as those that are used when parameter uncertainty is not a part of the decision-making environment.

A second line of research considers the consequences for interest rates of model uncertainty, and find that the policy maker is likely to be more responsive to changes in the economy rather than less when this form of uncertainty is taken into account. Another line has found that policy makers, faced with uncertainty about the data they have available, either because of measurement issues or the possibility of future revisions, will tend to act more cautiously. The feature that distinguishes data and model uncertainty from parameter uncertainty is that there is no straightforward way of characterising these types of uncertainty with a unique probability distribution. Following the literature, we label this Knightian uncertainty.

An important implication of allowing for the possibility that the future cannot be characterised by a single probability distribution is that the underlying behaviour of the policy maker can no longer be represented by preferences that generate standard expected utility results. This paper discusses the implications of Knightian uncertainty for the nature of the monetary policy maker's decision problem and the likely consequences this has for the observed path of interest rates. We conclude that an appropriate formulation of the decision-making process in the presence of Knightian uncertainty has the potential to explain the inertia in the level of the interest rate observed in actual interest rate paths.

The rest of this paper is structured as follows. In Section 2, we review the current body of literature that explores the effects of different forms of uncertainty on the decision-making behaviour of monetary policy makers. In Section 3 we discuss the distinction between risk, which can be thought of as uncertainty characterised by a unique probability distribution, and uncertainty, where such a characterisation is not possible. This is followed in Section 4 by a discussion about the two existing approaches to modelling decision-making under Knightian uncertainty. In Section 5 we discuss the applicability of each of these approaches to the monetary policy decision-making problem and in Section 6, we discuss how the standard tools of optimal control can be adapted to allow us to begin to operationalise the form of Knightian uncertainty we argue is more applicable in the context of monetary policy. We also show simulations of a simple closed economy model to formalise the intuition presented in Section 5. We conclude in Section 7 by assessing the value of thinking about monetary policy decision-making from a microeconomic decision theory perspective and discuss potential directions for future research.

2. The Current Literature

The optimal policy problem, when the economy evolves according to a linear difference equation and the monetary policy maker has quadratic preferences, can be written in a general form as:

$$\min_{\{i\}} Loss = E\sum_{t=1}^{N}\delta^{t-1}x_t'\Omega x_t \qquad (1)$$

subject to:

$$x_t = A + Bx_{t-1} + Cr_{t-1} + \Lambda_t \qquad (2)$$

where:

- ***x*** is a vector of deviations from policy targets, which generally includes deviations of inflation from the inflation target and output from potential;
- Ω summarises the preferences of the policy maker by assigning weights to each policy objective;
- ***i*** is the path of nominal interest rates;
- ***r*** is the path of real interest rates;
- δ is the discount rate;
- ***B*** describes the dynamic structure of the model;
- ***C*** describes how the economy responds to the policy instrument; and
- Λ captures additive shocks to the economy.

One of the puzzles that arises when estimating paths of optimal policy using this framework, is that the optimal path of interest rates is much more volatile and subject to policy reversals than paths that are typically observed (Lowe and Ellis 1997). Eijffinger, Schaling and Verhagen (1999) present a model in which policy changes involve small menu costs to explain why observed interest rates also exhibit prolonged periods of inaction. This paper is more closely associated with another direction of research that attempts to reconcile model generated and observed interest rate paths by considering the role of uncertainty.

Blinder (1998) suggests that actual interest rates may be less volatile than those predicted by these models because central bankers take into account the fact that they are uncertain about the models they are working with. Since this comment was made, the form of uncertainty that has received the largest amount of attention is parameter uncertainty.

The effects of parameter uncertainty were first raised by Brainard (1967) who shows, that in a simple static model of the macroeconomy, that adjustments to the policy instrument will be damped if the policy maker is uncertain about the parameters of the model. However, this result does not generalise to the dynamic case. Shuetrim and Thompson (1999) show that although uncertainty about the effect of the interest rate on output decreases the willingness of policy makers to change interest rates, uncertainty about the dynamic structure of the model can lead to the opposite result: policy makers may wish to change interest rates by more than in the absence of uncertainty. Similar theoretical results have been established by Söderström (1999).

Empirical work presented by Sack (1999) and Debelle and Cagliarini (2000) suggests that the first of these two effects dominates, and that the path of optimal policy does appear to be less volatile when parameter uncertainty is taken into account. However, other studies, notably Rudebusch (1999) and Estrella and Mishkin (1999), do not find such convincing results. Although Sack and Wieland (1999) suggest that the differences in these results may be due to the degree of richness in the dynamic structures or to the number of 'uncertain' parameters considered, the general conclusion appears to be that parameter uncertainty is of secondary importance for explaining the differences between model generated and actual interest rate paths.

Another branch of research has considered the effects of more general model uncertainty on the path of interest rates generated by an optimal decision problem. Intuitively, allowing for model uncertainty can be thought of as formalising Blinder's (1998) suggestion:

Use a wide variety of models and don't ever trust one of them too much. … My usual procedure was to simulate policy on as many of these models as possible, throw out the outlier(s), and average the rest to get a point estimate of a dynamic multiplier path. This can be viewed as a rough - make that very rough - approximation to optimal information processing. (Pages 12-13.)

As this quote suggests, general model uncertainty, unlike parameter uncertainty, cannot be conveniently summarised by a single probability distribution. Therefore, it is not possible to use standard optimal control methods to solve for optimal policy.

To help solve this more complicated problem, both Sargent (1999) and Onatski and Stock (1999) make an additional assumption that the policy maker aims to minimise the losses involved with the worst-possible scenario, given the range of models that are being considered as possibilities. Intuitively, as uncertainty aversion increases, the number of models the policy maker wishes policy to be robust to should increase. A wider range of models will allow for the possibility that a worse worst-case scenario will occur.

Although Sargent (1999) and Onatski and Stock (1999) formalise model uncertainty differently, the general result in both cases is that the path of interest rates should be more volatile when model uncertainty is taken into account than when it is ignored. This runs counter to the original motivation for considering uncertainty in the monetary policy decision-making problem discussed earlier. We argue in Section 5, is that this outcome is the direct result of the decision rule used and the particular characterisation of Knightian uncertainty that underlies it.

The final form of uncertainty that has received some attention is data uncertainty. This form of uncertainty becomes particularly important when measuring the output gap. One source of uncertainty about the output gap is the fact that output data are frequently revised. Policy made on the basis of first-release data may not look optimal by the time the data have been revised. It is also possible that with the help of coincident indicators, which are not subject to the same measurement problems, policy makers could have a better view of the true state of the economy than the output data and consequently, their actions may look better considered as the data are revised. The second problem facing measurement of the output gap is that potential output is not directly observed, and there is little agreement as to how it should be estimated.

Orphanides (1998) examines the effects of measurement error on the performance of efficient rules. Orphanides compares data that were available at the time monetary policy decisions were made with ex-post data to evaluate the size and nature of measurement error in the United States. He then estimates the impact this data noise has on the behaviour of policy makers. He finds that in the presence of data noise, a central bank following an optimal Taylor rule will act more cautiously in response to output and inflation data than they would if they were certain that the data were correct. Similar results were found by Smets (1998) and Orphanides *et al*. (1999) using a more sophisticated macroeconomic model.

Similarly to model uncertainty, data uncertainty can also be captured in the error structure of the model. To show this, it is useful to be more specific by using a simple backward-looking model of a closed economy used by Ball (1997) and Svensson (1997):

$$\begin{aligned} y_t &= \lambda y_{t-1} - \beta(i_{t-1} - \pi_{t-1}) + \varepsilon_t \\ \pi_t &= \pi_{t-1} + \alpha y_{t-1} + \eta_t \end{aligned} \tag{3}$$

where y is the output gap;

π is the deviation of inflation from target;

i is the deviation of the nominal cash rate from its steady state level; and

$$\begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_\eta^2 \end{bmatrix} \right) \tag{4}$$

Assume that the output gap at time *t-1* is measured to be zero, but that we believe that the true output gap could be above or below our estimate by about 0.5 percent. This deviation can be captured by allowing the mean of ε_t to have a range of [-0.5λ, 0.5λ], and for the mean of η_t to have the range of [-0.5α, 0.5α]. The error in the output equation will also capture uncertainty about the level of the neutral real interest rate. Consequently, data uncertainty can be translated into a range of possible means of the error processes, and can be thought of as a specific case of model uncertainty.

In summary, there is a significant body of research that has investigated the way in which different forms of uncertainty are likely to affect monetary policy decision-making. Perhaps the best understood of these is the effect of parameter uncertainty. This is partly because it is relatively straight-forward to model decision-making in situations where the uncertainty can be characterised by a single probability distribution. Unfortunately, parameter uncertainty does not appear to be sufficient for explaining the observed smoothness of interest rates, although Sack (1999) suggests that parameter uncertainty does go some way in explaining the smoothness of the Fed funds rate. Both data and model uncertainty can be thought of as examples of Knightian uncertainty. In Section 4 we discuss two ways of formalising Knightian uncertainty and argue in Section 5 that the differences in these methods can explain the different results generated by model and data uncertainty.

3. Risk vs Uncertainty

Before discussing the ways in which Knightian uncertainty can be incorporated into decision-making problems, it is useful to be clear about its definition. One of the first economists to make a distinction between risk and uncertainty was Frank Knight (1933). Knight's interest in this subject was spurred by the desire to explain the role of entrepeneurship and profit in the production process. Knight held the view that profits accruing to entrepreneurs are justified and explained by the fact that they bear the consequences of the risks (uncertainties) inherent in the production process that cannot be readily quantified.

There is a fundamental distinction between the reward for taking a known risk and that for assuming a risk whose value itself is not known. It is so fundamental, indeed, that, as we shall see, a known risk will not lead to any reward or special payment at all. (Page 44)

Known risks arise in situations where outcomes are governed by physical laws, eg a dice roll, or the factors affecting future outcomes remain more or less constant over time, eg mortality tables. In these cases, past observations of the distribution of outcomes will be a good guide for the distribution of outcomes that can be expected in future. At some point, however, the processes that need to be forecast become sufficiently complex and interrelated with the outcomes of the decisions of other agents, that the past does not provide such reliable information about the likelihood of future events occurring.

The fact is that while a single situation involving a known risk may be regarded as "uncertain," this uncertainty is easily converted into effective certainty; for in a considerable number of such cases the results become predictable in accordance with the laws of chance, and the error in such prediction approaches zero as the number of cases is increased. (Page 46)

LeRoy and Singell (1987) summarise Knight's distinction between risk and uncertainty by defining uncertainty as a situation where no objective, or publicly verifiable, probability distribution exists. In situations where a single objective probability distribution does not exist, LeRoy and Singell argue that Knight's exposition is consistent with the idea that the decision-maker forms some subjective probability. When a single probability distribution is available, it is straightforward to evaluate the expected value of pay-offs to different actions. However, forming a unique subjective probability distribution may not be straightforward as the Ellsberg paradox helps to illustrate.

Suppose we have a box of 300 balls, 100 of which are red and the rest are blue and green in undisclosed proportions. A ball is chosen at random from the box. Suppose we are offered the choice of betting on whether a red ball or a blue ball would be selected. Which should we choose to gamble on? Now suppose we are faced with a different gamble. We have to choose between betting on whether the ball is not red or not blue. Which gamble do we select in this case?

In most instances, individuals will pick red and not red in response to these two questions (Kreps 1990 based on Ellsberg 1961). If red is selected in the first gamble, the participant has implicitly evaluated their subjective probability of getting a blue ball to be less than the probability of a red ball. The paradox here is that in order for the same individual to act rationally according to the axioms of expected utility theory when faced with the second choice, they should bet that a blue ball will not be chosen. The Ellsberg paradox highlights the fact that people prefer situations when the probabilities are known to those where they are unknown. Red may be chosen in the first gamble not because we believe the probability of getting a red ball is greater than a blue ball, but because we know the exact probability of getting the red ball.

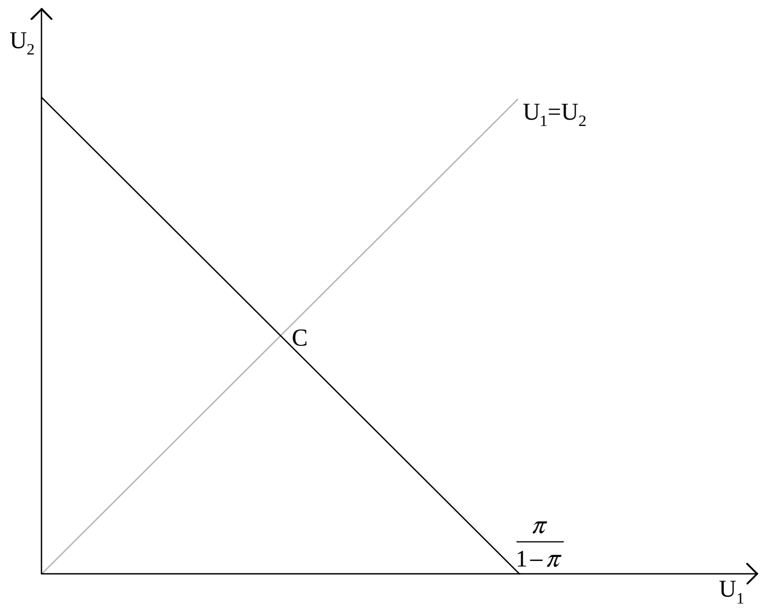
The question is, how should an individual make rational decisions when a range of probability distributions are possible? The Bayesian approach to this problem would be to decide on a probability distribution over the possible probability distributions, which essentially reduces the problem to one in which there is a single probability distribution. This solution assumes that the decision-maker is willing to make definite statements about the distribution of future outcomes. However, the interest in the problem arises from the fact that decision-makers are either unwilling or unable to make such a strong assumption about probabilities at the first stage of the decision-making process.

Another approach to is to recognise that all the axioms underlying expected utility theory may not apply in an environment where a range of probability distributions are possible. The corollary of this is that it will not be possible to use standard expected utility solutions to these decision problems. In the next section we consider the consequences for decision-making of relaxing the independence and completeness axioms. The implications of these changes for the conduct of monetary policy are discussed in Section 5.

4. Knightian Uncertainty and Expected Utility Theory

In cases where there is a single probability distribution over future events, decisions can be made by choosing the action with the maximum expected utility. Savage (1954) and Anscombe and Aumann (1963) provide an axiomatic foundation for this expected utility representation and therefore justify the use of the maximum expected utility decision rule.

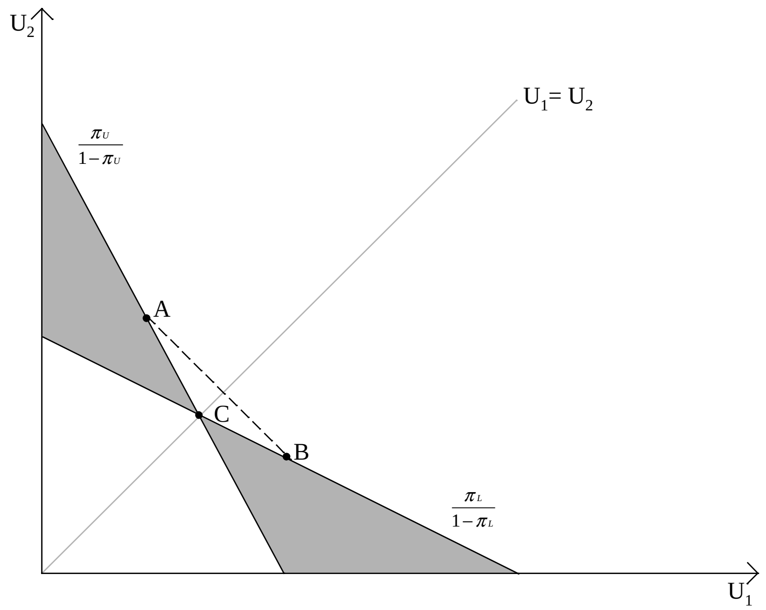
Figure 1



In what follows, we consider a simple decision problem in which there are two possible states of the world and decision-makers can hold assets that yield utility U_1 in state 1 and U_2 in state 2. Assuming that the unique probability of state 1 occurring is π, the combinations of U_1 and U_2 that yield the same expected utility will lie on a straight indifference curve with slope $\frac{\pi}{1-\pi}$, shown in Figure 1. With acts along the 45^{o} line, such as *C*, the decision-maker is certain of getting a particular level of utility regardless of the state of nature that occurs. These acts will be referred to as constant acts.

In the case of Knightian uncertainty, the decision-maker applies a range of subjective probability distributions over the possible states of nature. Assuming that the range of probabilities being considered for state 1 is continuous, the lower bound for this probability can be defined as π_L and the upper bound as π_U. The difference between π_U and π_L can be thought of as some measure of the degree of Knightian uncertainty facing the decision-maker. In Figure 2, the points that are indifferent to *C* with respect to a particular probability distribution in this range are represented by the shaded area. Note that the area shaded will become larger the higher the degree of uncertainty.[1]

Figure 2



[1] For a detailed discussion of aversion to uncertainty, see Epstein (1999).

Without a unique probability distribution, decisions can no longer be made by maximising expected utility: actions that maximise utility under some probability distributions in the range will not be optimal under others. With this in mind, there have been two main approaches to modelling decision-making under Knightian uncertainty.

The first approach relaxes the independence axiom and instead assumes that preferences are consistent with uncertainty aversion and a constant-independence assumption. The second approach drops the assumption that preferences are complete. We present each of these cases in turn and discuss the implications these changes have for the decision rules that should apply. The implications of these approaches for monetary policy making are discussed in Section 5.

4.1 Relaxing the Independence Assumption

The independence assumption has perhaps been the most controversial axiom of expected utility theory since it was formalised by Savage (1954) as the 'Sure Thing Principle'. The independence axiom states that if *A* and *B* are two gambles, and *A* is preferred to *B*, that is $A \succ B$, a mixture of *A* with a third gamble *D* will be preferred to a mixture of *B* and *D* in the same proportions. That is $\alpha A + (1-\alpha)D \succ \alpha B + (1-\alpha)D$, where α is strictly between 0 and 1. The indifference curves in Figure 1 are linear if and only if the independence axiom holds.

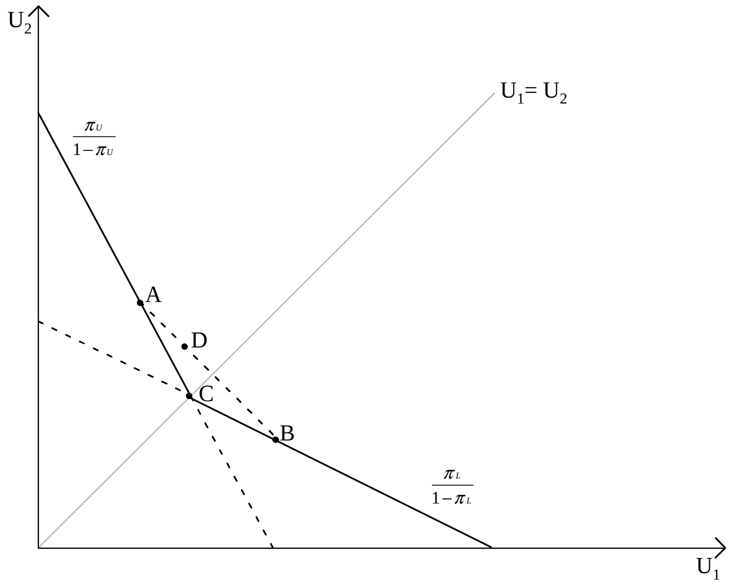
Gilboa and Schmeidler (1989) argue that the independence axiom will not hold in situations where there is no unique probability distribution, but that a weaker version of independence, which they label constant-independence, will hold. Constant-independence states that if $A \succ B$, then a mixture of *A* with a constant act, *C*, will be preferred to a mixture of *B* with the constant act *C*, that is $\alpha A + (1-\alpha)C \succ \alpha B + (1-\alpha)C$.

The other axiom Gilboa and Schmeidler introduce is uncertainty aversion that Schmeidler (1989) explains "means that 'smoothing' or averaging *utility* distributions makes the decision-maker better off." This property arises from the way in which the independence assumption has been relaxed. In situations where there is a single probability distribution, and independence holds, it will be true that if *A* is indifferent to *B*, that is *A*~*B*, then $\alpha A+(1-\alpha)B \sim \alpha B+(1-\alpha)B=B$. Gilboa and Schmeidler show that if the independence assumption is relaxed, it is possible that for $A \sim B$, that $\alpha A+(1-\alpha)B \succ B$ for α strictly between 0 and 1 (see below).

Using the weaker definition of independence and introducing uncertainty aversion, Gilboa and Schmeidler (1989) prove that this description of preferences provides a theoretical foundation for Wald's (1950) minimax decision rule. This rule formalises the idea that in situations where there is uncertainty about the probability distribution to apply, decision-makers may choose to minimise the impact of the worst case scenario.[2] More formally, Gilboa and Schmeidler specify the preference relationship between two gambles, *A* and *B* as:

$$A \succ B \text{ if and only if } \min_{\pi} E_{\pi}U(A) > \min_{\pi} E_{\pi}U(B).$$

Figure 3: Gilboa-Schmeidler Preferences



This decision rule allows points that are indifferent to a given constant act, *C*, to be represented by the indifference curves presented in Figure 3. The transition from Figure 2 to Figure 3 can be explained as follows. Above the 45° line, utility of state 2 is greater than the utility of state 1. The minimum expected utility will be

2 For monetary policy decision-making, this is similar to minimising the maximum expected loss.

obtained by applying the lowest probability for state 2, that is the highest probability of state 1. Hence, the appropriate slope of the indifference curve above the 45° line is $\frac{\pi_U}{1-\pi_U}$. Using the same argument, the slope of the indifference curve satisfying the minimax rule below the 45° line is $\frac{\pi_L}{1-\pi_L}$. The consequence of the minimax decision rule, therefore is that Gilboa - Schmeidler preferences are represented by kinked indifference curves, where the kink occurs on the 45° line.

The region above the curve contains elements strictly preferred to elements on the indifference curve since each element above the curve gives a higher minimum expected utility. The reverse is true for the area below the curve.

Constant-independence will hold in Figure 3: given that *A~B* it is true that

$$\alpha A+(1-\alpha)C \sim \alpha B+(1-\alpha)C$$

when *C* is a constant act that lies on the 45° line. To understand how these preferences embody uncertainty aversion, we can see that the decision-maker is indifferent between *A* and *B*. However, any linear combination of *A* and *B*, such as *D*, is preferred to both *A* and *B*. This is to say, that *D* will yield a higher minimum expected utility than either *A* or *B*, despite the fact that *A* and *B* will give the decision-maker the same minimum expected utility.

4.2 Relaxation of Completeness

An alternative approach to characterising preferences and describing the appropriate decision rule in situations where more than one probability distribution is possible is to consider maintaining the independence assumption, but relax the completeness assumption. Completeness implies that the decision-maker faced with two gambles will be able to compare them and state a preference relation between them.

Gilboa-Schmeidler preferences are complete because the decision-maker is willing to specify a single probability distribution to apply to any given alternative. If, however, a decision-maker thinks that it is important to consider alternatives under the full range of possible probability distributions, it is not possible to reduce the points indifferent to a given constant act to a linear, albeit kinked, indifference curve. Therefore, some points may not be comparable. The possibility of capturing the effects of Knightian uncertainty by dropping the completeness axiom was introduced by Bewley (1986).[3]

One of the implications of dropping completeness is that it is possible for decision-makers to make a series of decisions that are not consistent with each other. In order to prevent such intransitivity, Bewley introduces the inertia assumption. This assumption states that the decision-maker remains with the *status quo* unless an alternative that is clearly better under each probability distribution considered is made available. This can be thought of as a form of caution: if you are uncertain about whether you will be better off with the alternative, why change your current position?

Bewley preferences imply that faced with two alternatives, *X* and *Y*, *X* will be preferred to *Y* if and only if $E_\pi U(X) > E_\pi U(Y)$ for all π considered. That is to say only when *X* yields better expected outcomes compared to *Y* under every distribution will *X* be preferred to *Y*.

More generally, it can be shown that any point in the shaded region of Figure 4 will not be dominated by the *status quo*, *Z*. For example, a point such as *W* is better than *Z* under some probability distributions but worse than *Z* under others. In this case, the decision-maker does not have any clear preference for either *Z* or *W* but because of the inertia assumption, the decision-maker will remain with *Z*. Point *V* is clearly better than *Z* since it gives a higher expected utility under every probability distribution. Should *V* become available, the decision-maker would choose *V* over *Z*. The diagram also indicates that by mixing two acts, both incomparable to *Z*, an act that is comparable to *Z* can be produced. So, although the decision-maker will not neither *W* nor *Y* over *Z*, the decision-maker will prefer a mixture of *W* and *Y* to *Z*. The willingness of the decision-maker to make a decision will depend on how much uncertainty exists. The less uncertain the act, that is, the closer are π_L and π_U, the larger the set of acts that are directly comparable to the *status quo*, and the more likely the decision-maker will be more willing to give up the *status quo* for an alternative.

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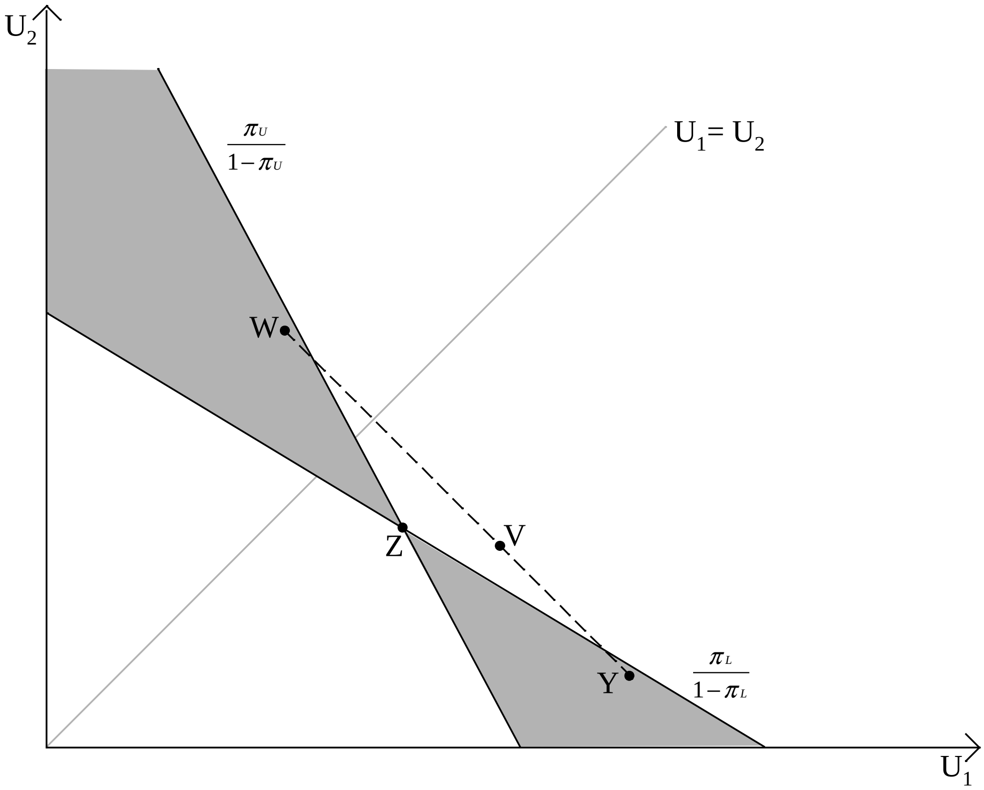
[3] In fact, Bewley asserts that, in his framework, preferences will be complete if and only if there is only one subjective probability distribution.

Figure 4: Bewley Preferences



Although the inertia assumption gives the decision-maker direction when the decision-maker is faced with incomparable alternatives, it does not provide guidance in situations where there is more than one alternative that strictly dominates the current position. There are a number of possible decision rules that could be used, ranging from a rule such as minimax, to rules of thumb such as picking the mid-point of the range.

5. Implications for Monetary Policy

The two alternative approaches to dealing with situations in which there is more than one probability distribution presented above have different underlying assumptions about the preferences of policy makers and the decision rules that they use. In this section we discuss the applicability of these two specifications to the monetary policy decision-making environment.

5.1 Gilboa-Schmeidler Preferences

A decision-maker with Gilboa-Schmeidler preferences is assumed only to care about the probability distribution that delivers the worst case scenario, and chooses the action that minimises the downside of that worst case.

The main application of Gilboa-Schmeidler preferences has been in finance where Knightian uncertainty has been used to explain the behaviour of prices in equilibrium. For example, Epstein and Wang (1994) adapt the static Gilboa-Schmeidler framework to an inter-temporal setting to show that when uncertainty exists in a pure exchange economy, asset prices are indeterminate within some range defined by the degree of uncertainty. Therefore, for a given set of fundamentals, a continuum of prices may exist in equilibrium. The results Epstein and Wang (1994) generate lead them to concur with the view of Keynes (1936) that "existing market valuation…cannot be uniquely correct, since our existing knowledge does not provide a sufficient basis for a calculated mathematical expectation."

Sargent (1999) and Hansen, Sargent and Tallarini (1998) and Onatski and Stock (1999) have used robust control methods to make Knightian uncertainty and the minimax rule implied by Gilboa-Schmeidler preferences operational in dynamic models. Robust control methods provide a means of finding the actions that will be robust to a given range of possible outcomes, or deviations from the model being used.

Onatski and Stock (1999) specify a model in which the policy maker takes into account the fact that the true linear constraints that define the dynamics of the economy may differ in a general way from the linear constraints they are using to make their decisions. They also simplify the decision problem by assuming that monetary policy makers use a simple Taylor rule rather than a fully optimal rule for setting interest rates. Therefore, in this framework, the policy maker aims to choose the weights of variables within this rule to minimise the loss function under wost-case scenarios.[4] Onatski and Stock's (1999) model is very general, and consequently numerical methods must be used to find solutions. They note that:

Because the results are numerical, however, it is difficult to develop much intuition or to assess their generality

To the extent that it is possible, their general result is that the interest rate paths generated when model uncertainty is taken into account are more volatile than when there is assumed to be no uncertainty. This result runs counter to the original motivation for investigating the consequences of different forms of uncertainty for monetary policy decision-making. This result has also been found by other papers using robust control methods in the context of monetary policy decision-making. In each case, the result is sensitive to the range of allowable models used.

Sargent (1999), which follows earlier work by Hansen, Sargent and Tallarini (1998) and Hansen, Sargent and Wang (1999), also uses robust control methods to investigate the effects of model uncertainty on monetary policy decision-making. By using a different specification for the way in which the true model can deviate from the model being used, Sargent is able to introduce uncertainty averse behaviour via an extra term in the objective function that includes a 'preference for robustness' parameter. If the preference for robustness is set to zero, the standard optimisation problem will be obtained. When the preference for robustness is not zero, the decision-maker's behaviour will approximate the minimax decision rule. As this preference increases, the larger the set of possible deviations from the true model the policy maker wishes to consider, and the worse the worst-case scenario can be. This leads to the result that monetary policy makers who have a higher preference for robustness will have more volatile interest rate paths, as they try to reduce the losses that would occur under the worst-case scenario.

Robust control methods implement the Gilboa-Schmeidler formulation of the monetary policy decision-maker's problem by using the minimax rule. However, there are several reasons why other methods of incorporating Knightian

[4] It is also interesting to note that Onatski and Stock (1999) include the squared difference in interest rates in their objective function to reduce the volatility of the interest rate paths generated by the model.

uncertainty should be explored. First and foremost, the intuition for using a minimax rule in the context of monetary policy does not accord with the way in which monetary policy makers talk about their decisions. Monetary policy makers are more likely to talk in terms of the balance of risks rather than in terms of avoiding worst-case scenarios. Second, it is difficult to generalise the results of these models, and to the extent that this is possible, they deliver the result that interest rates will be more volatile in the presence of uncertainty than without it. This runs counter to the original motivation for looking at the effects of uncertainty on monetary policy decision-making.

5.2 Bewley Preferences

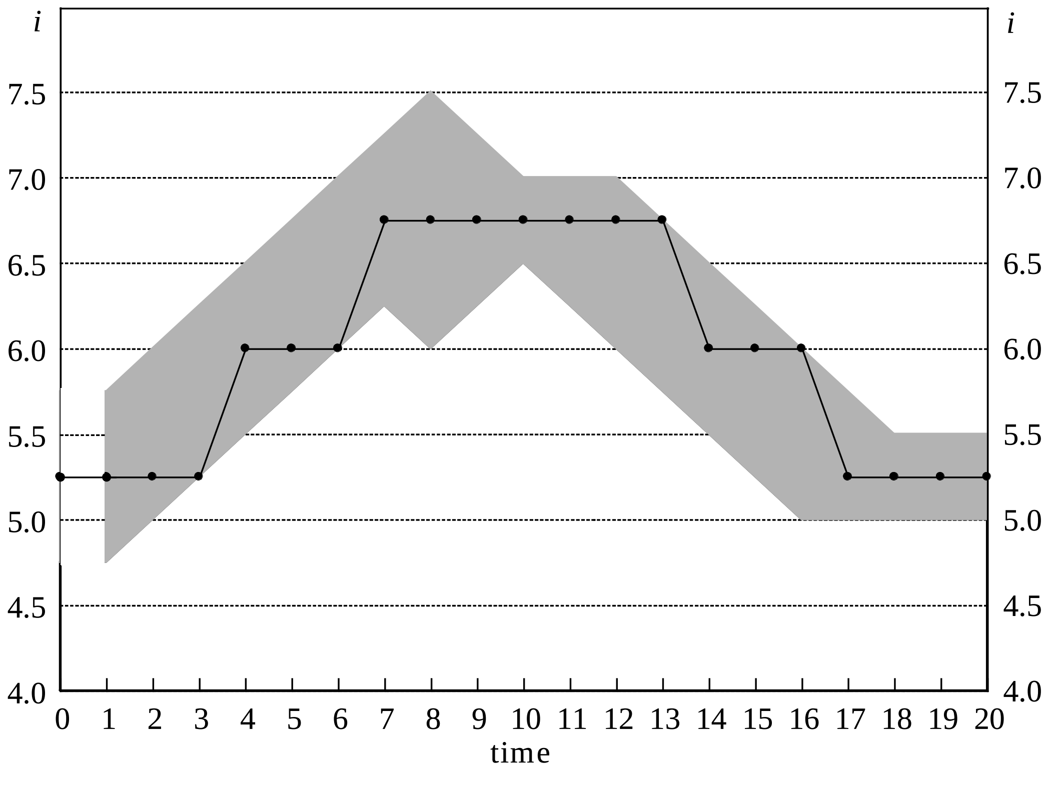
Unlike Gilboa-Schmeidler preferences, Bewley preferences assume that the policy maker cares about the outcomes under all the probability distributions they believe are possible, and do not focus solely on the worst-case scenario. The idea that policy makers facing Knightian uncertainty cannot make sharp comparisons between some alternatives is intuitively appealing, and accords more closely with the balance-of-risks approach to making monetary policy decisions.

Given the expectations about the ranges of probability distributions that are possible over future periods, it is theoretically possible to establish a range of interest rates that are consistent with these expectations. The inertia assumption states that the policy maker should leave interest rates unchanged if the current interest rate lies within this range. Consequently, it is possible that the current level of the nominal interest rate will persist for several periods if the policy maker's belief is that the level of the output gap and the real interest rate will remain in similar ranges for several periods. Therefore, the Bewley formulation of Knightian uncertainty has the potential to explain one of the stylised facts of observed interest rates.

Another stylised fact, is that interest rates tend to move in sequences of discrete steps in the same direction. Sack (1999) has demonstrated that the observed sequences of interest rate movements can largely be explained by the inherent autocorrelation in the variables that are included in the policy maker's objective function. Intuitively, the Bewley formulation of Knightian uncertainty can help to explain why these movements might be discrete.

If the policy maker with Bewley preferences has beliefs about the output gap and the neutral real interest rate that suggest the current stance of monetary policy is no longer within the appropriate range, there will be a discrete shift in the level of the interest rate. Bewley (1986, 1987) does not discuss what the decision rule in this sort of situation should be. Appropriate decision rules would include rules such as the minimax rule discussed earlier, as well as rules of thumb such as picking the mid-point of the range of possible interest rates, or the interest rate closest to the current level. Figure 5 demonstrates the way in which interest rates would be expected to move if the policy maker is faced with a given range of interest rates at each point of time.

Figure 5



The shaded region is an example of the way in which the range of appropriate interest rates might evolve. In Section 6 we discuss some of the issues that arise for deriving such ranges, and take them as given in Figure 5. At each point in time the policy maker must decide which interest rate to choose for the next period given their current interest rate and the range of appropriate interest rates for the next period. In Figure 5, a simple decision rule is used whereby the policy maker does not change the interest rate if the prevailing rate is in the optimal range in the following period. If the prevailing rate is not in the optimal range, the policy maker changes the interest rate to the midpoint of the range.

Figure 5 exhibits the step movements in interest rates, and the way in which autocorrelation in the macroeconomic variables in the policy maker's objective function can lead to sequences of interest rate movements in the same direction. Although this intuitive explanation of the way in which interest rates are likely to move when the policy maker has Bewley preferences and is using a combination of the inertia assumption and a mid-point decision rule, the difficulty in moving forward is determining how the ranges of interest rates should be determined. We begin to explore this issue in Section 6.

6. The Optimal Control Problem with Bewley Preferences

Following the discussion above, we would like to find an method of incorporating Bewley preferences into an optimal control framework. At a very general level, we want to minimise the policy maker's loss function subject to the linear constraints that define the dynamics of the economy, the range of expectations the policy maker has about the distributions of future shocks and the decision rules that are being used. By taking into account all these features, we should obtain internally consistent future paths of interest rates, that is the observed path of interest rates should not be different to the predicted path if no unexpected events occur.

When there is a single probability distribution over future shocks, it is relatively straightforward to obtain the unique, internally consistent path. However, when there is more than one probability distribution, it is much more difficult to obtain this internal consistency, and the uniqueness of the path is not guaranteed. The difficulty can be illustrated through the use of a two period problem where the Bewley policy maker must select nominal interest rates for the next two periods, i_1 and i_2, given their range of subjective expectations.

The policy maker could begin by choosing an arbitrary value for $i_2 = \hat{i}_2$. Given the current state of the economy, the range of subjective expectations and the choice of $\hat{i}_2$, optimal control techniques can be applied with each subjective probability distribution in turn, to generate a range for i_1. By applying the decision rule, we can determine the choice of i_1. We will denote this choice $\hat{i}_1$. Taking this choice of $\hat{i}_1$ and the current state of the economy and expectations, we can calculate the range of optimal values for i_2. By applying the decision rule, we can determine the choice of i_2 given $\hat{i}_1$, denoted $\tilde{i}_2$. If $\tilde{i}_2 = \hat{i}_2$, then a solution has been found. If not, we can iterate on the procedure until a solution is found.

This sketch of the Bewley policy maker's problem in a two period setting would be made more concrete by resolving existence and uniqueness issues and proving that if a solution exists, it is internally consistent. It is also clear that it is not straightforward to extend this solution method to the multi-period case as the sequential nature of decision-making is important. The conditions for existence, uniqueness and internal consistency are straightforward in the multi-period case when there is a single subjective probability distribution.

As a first pass at this problem, we propose an algorithm that establishes the range of feasible interest rates for the next period, taking into account the range of expectations over future shocks, but ignoring the decision rules that determine the given interest rate that will be chosen in each period. In Section 6.2, we simulate interest rate paths with this algorithm and show that despite the simplicity of the model and the fact that we are not using a fully optimal solution, we obtain evidence that this method generates interest rate paths that replicate some important features of observed interest rates.

6.1 A Solution Algorithm

In the general model presented in Section 2, the monetary policy decision-maker chooses interest rates to minimise the loss function in Equation (1). In a finite *N* period problem, the future path of interest rates can be written as the solution to *M=N-(number of interest rate lags in the output equation)* first order conditions:

$$\frac{\partial Loss}{\partial i_1} = 0 \qquad (1)$$
$$\vdots \qquad \vdots$$
$$\frac{\partial Loss}{\partial i_M} = 0 \qquad (M)$$

In the case where there is a single probability distribution, this will yield a unique path of interest rates. In the case where there is a set of probability distributions, this will not be possible. However, it is possible to derive a value for i_1 from a given probability distribution from this range assuming, counterfactually, that future interest rate will be determined as optimal outcomes from this probability distribution. If the set of probability distributions is convex, the solution to this continuum of problems will yield a range of possible interest rates for i_1. In fact, given convexity, it is possible to find the range by solving for the upper and lower bounds. Once a range for i_1 has been determined, the relevant decision rules can be applied to choose the specific interest rate from the range.

6.2 A Simulation of Monetary Policy in a Simple Model

The model used for this simulation is the small closed economy model used by Ball and Svensson discussed in Section 2. We have calibrated the parameters of the model in a similar fashion to Ball (1997). This annual model is described below:

$$\begin{aligned} y_t &= 0.8y_{t-1} - 0.8(i_{t-1} - \pi_{t-1}) + \varepsilon_t \\ \pi_t &= \pi_{t-1} + 0.4y_{t-1} + \eta_t \end{aligned} \qquad (5)$$

An increase in the *ex-post* real interest rate by 1% in a given year reduces the level of the output gap by 0.8% in the following year. No long-run trade-off exists between inflation and output in this model and the implied sacrifice ratio is equal to 2.5 per cent years.

Knightian uncertainty is introduced into the framework by allowing the policy maker to consider a range of possible distributions for the shocks to the model by allowing for a range of different means for these distributions. To ensure convexity, we assume that these means lie in a continuous bounded range.

Orphanides (1998) thoroughly examined the effects of uncertainty about the level of the output gap and inflation on monetary policy decision-making. In this study, we only consider the effect of uncertainty surrounding the measurement of the output gap. Although uncertainty about the level of the neutral real interest rate is closely linked to measurement issues with the output gap, we ignore this potentially important source of uncertainty to simplify the simulation.

In particular, we assume that the policy maker is uncertain about last year's output gap to within one per cent. This implies that the mean of the errors in the output equation could range between –0.8 per cent and +0.8 per cent.[5] We assume that the range of the means represents two standard deviations of the error distribution to ensure that the Knightian uncertainty is 'significant'. This would imply that one standard deviation would equal 0.8 per cent. Similarly, for inflation shocks, the mean of the errors would range from -0.4 to +0.4 per cent, and this would imply that one standard deviation for the inflation shocks is 0.4 per cent.

Two simulations are performed. The first simulation is a standard optimal policy exercise that assumes the output gap can be measured precisely. The second simulation assumes that the policy maker recognises that the measurement of the output gap is imprecise. This is the case of Knightian uncertainty, and we assume that the policy maker chooses the mid-point of the range when the inertia assumption does not apply. Stochastic shocks were generated independently using normal distributions with a zero mean and the relevant standard deviation. We assume that the policy maker chooses the nominal interest rate at the beginning of the period, and observes the output gap and inflation at the end of the period, at which time they 're-optimise' to select the nominal cash rate for the following period.

The results of these two simulations are presented in Figures 6 and 7. Although both are quite volatile, when Knightian uncertainty is taken into account there are extended periods of time where the nominal interest rate does not move.

[5] $\varepsilon_t = \bar{\varepsilon}_t + \nu_t$ and $\eta_t = \bar{\eta}_t + \omega_t$ where ν and ω are mean-zero stochastic shocks. $\bar{\varepsilon}$ is the mean of the output shock, assumed to lie in the range –0.8 to +0.8. Likewise for $\bar{\eta}$.

Figure 6: Simulated Path of Interest Rates

Optimal policy without Knightian uncertainty

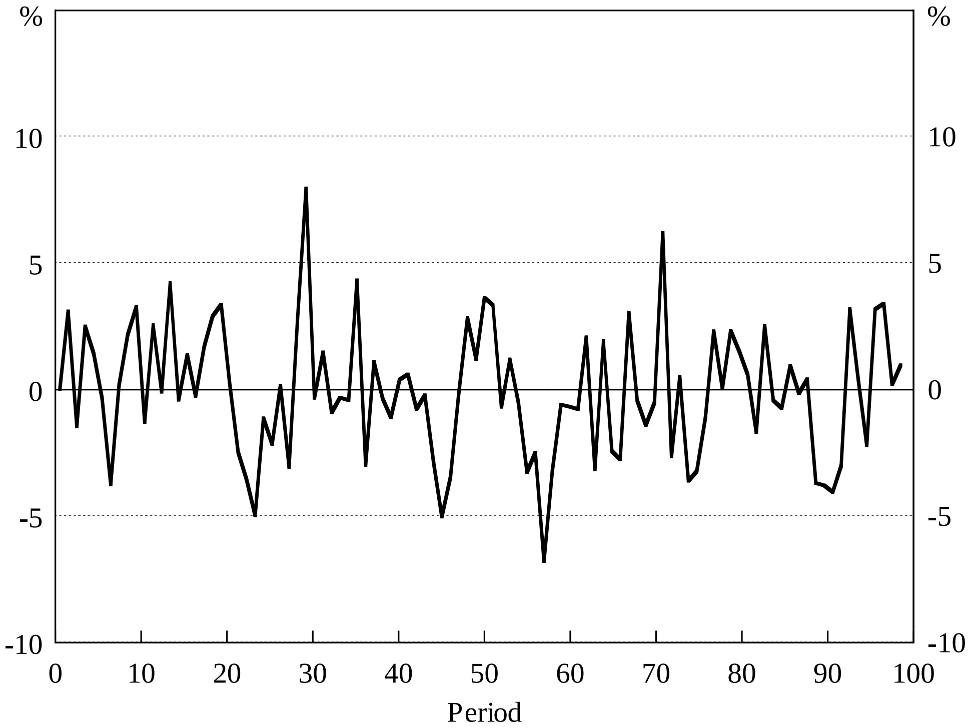
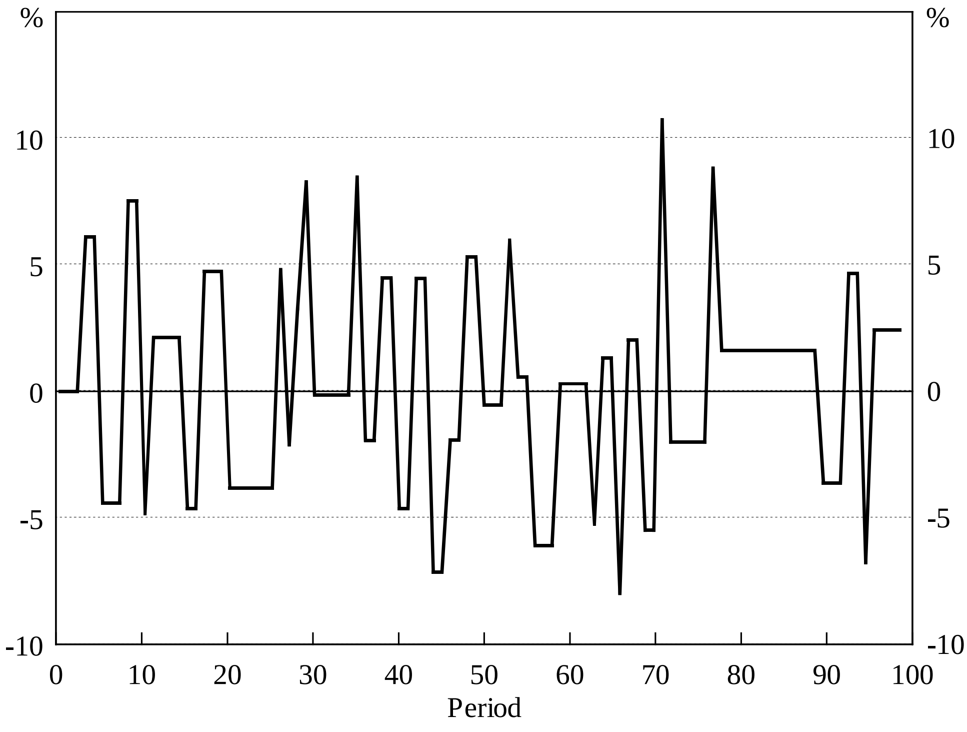


Figure 7: Simulated Interest Rate Path

Including Knightian Uncertainty



As a point of comparison, we present the results of these simulations with a penalty for volatility in one simulation and a penalty for policy reversals in the other. The difference between these simulations is the objective function. By including the term $(i_t - i_{t-1})^2$ in the objective function, as is often used in the literature, the policy maker is penalised for moving the interest rate too much. That is, there is less of a penalty in moving the interest rate from 1% to 2% than moving the interest rate from 1% to 3%. It should be noted that it is only the movement in the interest rate that is penalised and not its direction, which leads us to consider a second kind of penalty in the objective function. The results of penalising volatility are presented in Figures 8 and 9.

In general, the changes in interest rates in these two simulations are smaller than when volatility is not penalised. The simulation that takes into account Knightian uncertainty exhibits more step-like behaviour and there are fewer immediate policy reversals.

If the policy maker has moved the interest rate in one direction in a given period, they may not wish to reverse the decision in the very next period. The policy maker may wish to avoid reversing policy if this leads to a loss of credibility or may make the signal to market participants about the stance of monetary policy less clear. An aversion to policy reversals can be captured by including the term $(\Delta i_t - \Delta i_{t-1})^2$ in the loss function. This term penalises large differences in the direction in interest rate changes in two subsequent periods. The results of these simulations are presented in Figure 10.

Figure 8: Simulated Interest Rate Path

Optimal policy with volatility penalty without uncertainty

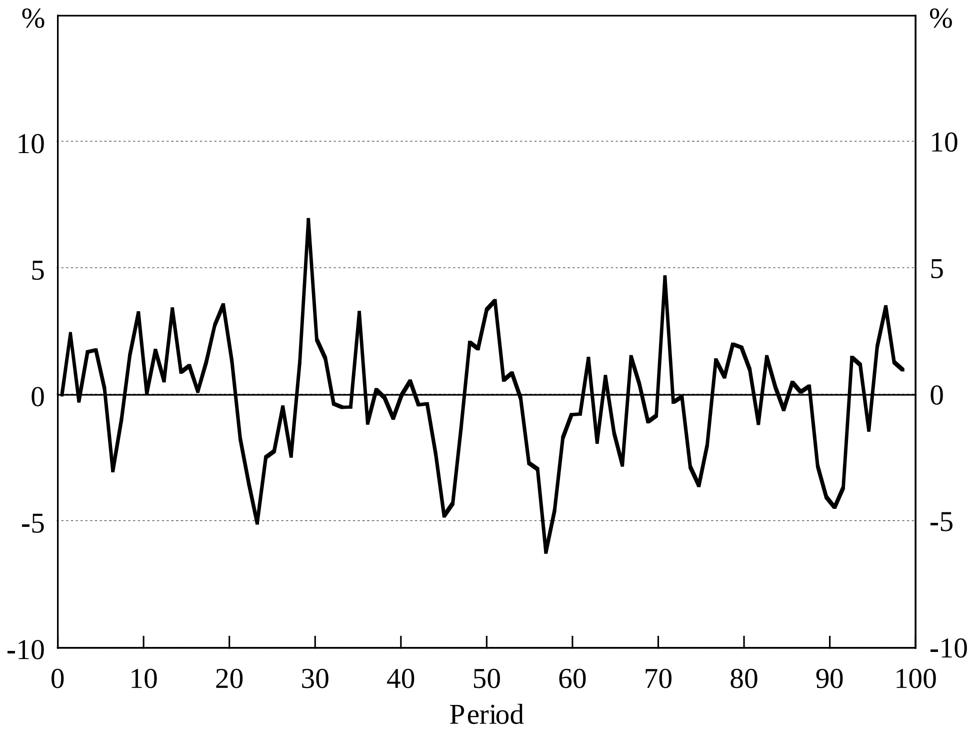


Figure 9: Simulated Interest Rate Path

Including Knightian uncertainty with volatility penalty

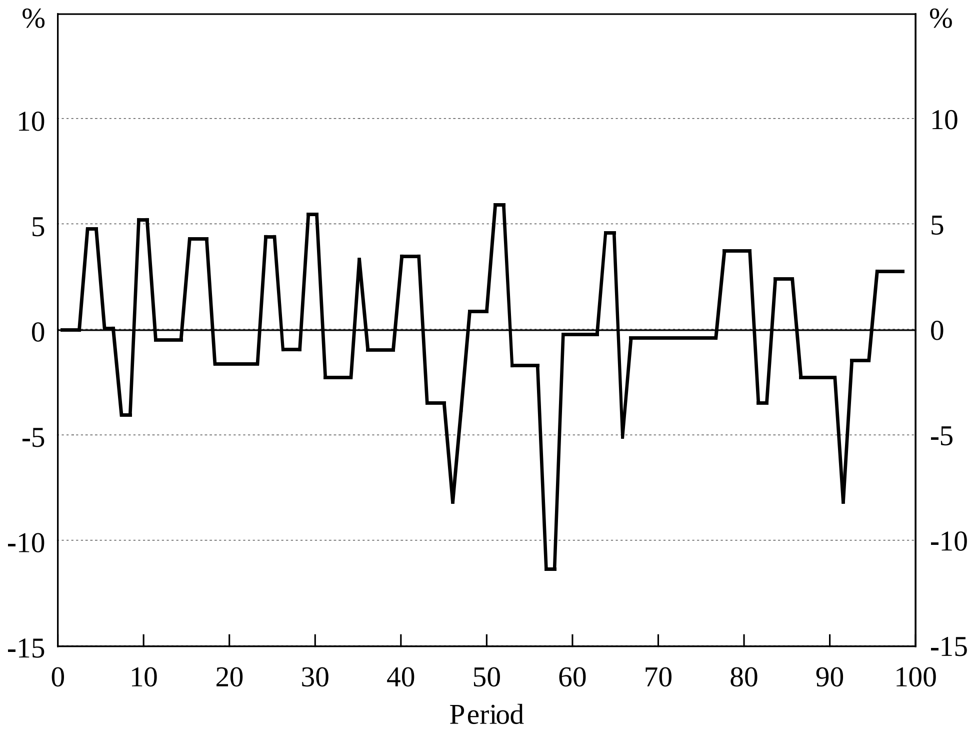
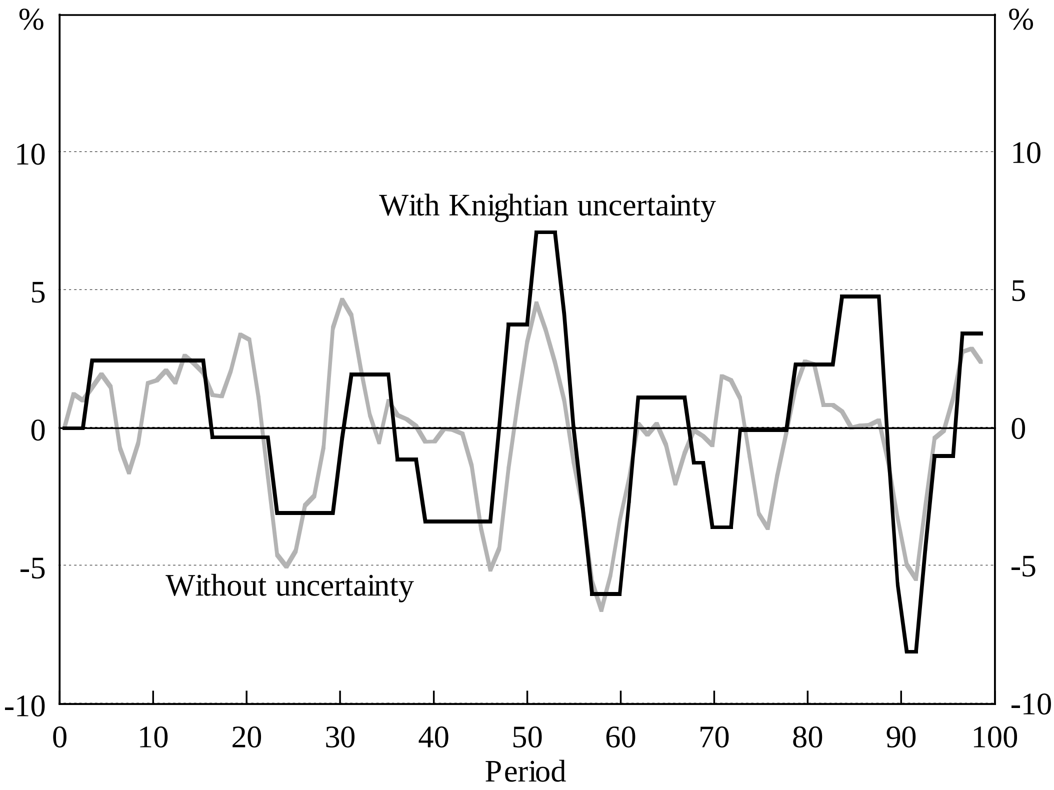


Figure 10: Simulated Interest Rate Paths

With policy reversal penalty



As expected, both paths of interest rates appear much smoother. Interestingly, although the penalty for policy reversals has the same weight in both cases, there are fewer reversals in the Knightian uncertainty simulation, and several periods of no change tend to lie between changes in direction as observed in practice. Despite the simplicity of the model and the fact that the algorithm is not fully optimal, the path of interest rates under Knightian uncertainty, with some penalty for policy reversals, is strikingly similar to interest rate paths observed in practice.

7. Conclusions and Further Research

This paper has explored the way in which Knightian uncertainty can help us reconcile interest rate paths from optimal policy problems with those observed in practice. We do this by providing a framework for characterising different types of uncertainty and demonstrating that the increased complexity associated with allowing for Knightian uncertainty has the potential to generate paths of interest rates closer to those observed in practice with a simple simulation exercise.

Discussions of model and data uncertainty in the monetary policy literature implicitly make use of Knightian uncertainty. Recent analyses of model uncertainty have explicitly used minimax decision rules, which directly result from one particular formalisation of Knightian uncertainty. We argue that the implied desire of monetary policy decision-makers to avoid worst-case scenarios does not accord with how monetary policy makers talk about their decision-making.

We advocate an alternative formalisation of Knightian uncertainty due to Bewley (1986) that assumes the policy maker wishes to compare alternatives under all possible probability distributions in order to make their decisions. Intuitively, this formalisation provides monetary policy decision-makers with a range of viable interest rates which, when combined with additional decision rules, can generate paths of interest rates with extended periods of no change.

Using a simple closed economy model and assuming that imprecise measurement of the output gap as a source of Knightian uncertainty, we use a simple algorithm based on the Bewley formalisation of Knightian uncertainty to generate interest rate paths. These paths exhibit periods of no change and, with some smoothing, are strikingly similar to those observed in practice.

Although the results of our simulation are promising, there are several issues that warrant further research. Perhaps most importantly, the algorithm we have proposed is not fully optimal. Another direction of further research is to explore more sophisticated models that contain more realistic lag structures and, perhaps, incorporate a richer set of explanatory variables. We believe extensions along these lines will be fruitful.

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