

Monetary Policy and Fiscal Foresight

Nora Traum*

Abstract

Legislative and implementation lags inherent in the political process often allow private agents to receive news about their future tax rates, a phenomenon known as fiscal, or more specifically tax, foresight. This paper investigates the effects of fiscal foresight on monetary policy under various assumptions about the information set of the monetary authority. We examine the welfare effects of tax foresight and how such foresight affects the monetary authority's ability to implement its policy. Our model is a simple version of the dynamic sticky price models extended to include possible foresight about changes in distortionary taxes. We find that the optimal response of the interest rate to anticipated tax changes can be qualitatively different from the response under a simple Taylor rule and that welfare rankings under foresight depend crucially on the welfare measure. In addition, identification issues present when the monetary authority has partial information about the state of the economy make it impossible for the monetary authority to recover the correct structural disturbances from its observable variables. This, in turn, causes the monetary authority to induce history dependence in endogenous variables and worsen the welfare of private agents.

1 Introduction

Information about when and how taxes will change is often released in advance of an actual tax alteration, giving agents “fiscal foresight” or foreknowledge about future fiscal adjustments. Such foresight can result from two types of lags that mainly are attributed to the political process: a “legislative lag” between when a tax law is proposed and when it is passed and an “implementation lag” between when the legislation is signed into law and when it actually takes effect. The length of these lags can vary depending on the particular legislation considered. Yang (2007a) documents that the average implementation lag alone is seven months in the postwar U.S. history. Nonetheless, it is not unusual for economic models to assume that changes to fiscal policy are *unanticipated* by the private sector and the monetary authority. While economists do not doubt the presence of tax

*Date: September 24, 2008. Department of Economics, Indiana University, ntraum@indiana.edu. I thank Eric Leeper and Todd Walker for many helpful comments and suggestions. In addition, I thank Tim Cogley, Juan Carlos Escanciano, and Bruce McGough for helpful conversations. Any remaining errors are my responsibility.

foresight, its theoretical implications remain largely unexplored. This paper investigates one such area, fiscal foresight's influence(s) on monetary policy, by seeking to characterize the welfare effects of tax foresight and how such foresight affects the monetary authority's ability to implement its policy.

Previous theoretical and empirical research on fiscal foresight is limited. Using a narrative approach to identify U.S. tax changes, Romer and Romer (2007) and Mertens and Ravn (2008) find that unanticipated and anticipated tax changes have different effects on output and investment. Theoretical results of Mertens and Ravn (2008) and Yang (2005) seem consistent with these empirical findings. These results suggest that foresight has nontrivial consequences and that it is important for understanding movements in aggregate variables. Others¹ also have found evidence for tax foresight and documented some of the theoretical consequences of its presence. This paper extends the theoretical analysis of fiscal foresight to a simple New Keynesian model and examines the effects of foresight under various assumptions about information set of the monetary authority. Specifically, this paper addresses two questions:

- What is the optimal monetary policy response to unanticipated and anticipated tax rate changes?
- If the central bank's information set is limited, how does tax foresight affect its ability to achieve its optimal monetary policy?

We investigate these issues in a simple Calvo pricing model extended to include possible foresight about changes in distortionary taxes. We find that anticipated tax rate changes lead to a higher unconditional welfare loss than unanticipated changes because news of a tax rate change increases the variances of endogenous variables by introducing moving average components in the solution paths of variables. However, we find the additional welfare loss due to the presence of fiscal foresight is quite small. In addition, we find that conditional on the history of disturbances, anticipated tax rate changes can be welfare improving because foresight gives agents more information about the expected value of future endogenous variables and this informational gain can improve the agents' welfare. Also, we find that in the presence of fiscal foresight, some simple, "implementable" rules (specifically certain Taylor rules²), often result in the monetary authority responding to fiscal news by moving the interest rate in a direction *opposite* to that under the optimal monetary policy.

We then consider the effects of monetary policy when there is a particular type of asymmetry between the information available to the central bank and that available to the private sector. In order for the central bank to implement its monetary policy, it needs to know the realizations of

¹Notable examples include Branson, et. al. (1985), Poterba (1988), Leeper (1989), Yang (2007b), and Leeper, Yang, and Walker (2008).

²See Schmitt-Grohe and Uribe (2004a, 2007) for examples of so-called simple, "implementable" monetary policy rules.

the current structural disturbances. In a real economy, it is very difficult for the central bank to directly observe such variables. Furthermore, it is difficult for the central bank to convert news about future tax rates into specific values by which the tax rate will move in the future. Although central bankers read newspapers and receive information about future tax rate changes, it can be hard for them to pinpoint the exact amount and nature by which aggregate tax rates change. For these reasons, we assume that the central bank only observes a subset of all economic variables and does not observe the underlying history of structural shocks or fiscal news in the economy. This complicates matters for the central bank in that the monetary authority must estimate these disturbances in order to implement its policy. In contrast, we assume that private agents have complete information about the current state and history of the economy. Thus, the information set of the private agents is strictly larger than that of the central bank.

We assume that the central bank uses the Kalman filter to optimally extract information about the structural disturbances from its observables. We follow the method of Svensson and Woodford (2004) to solve for the monetary authority's optimal policy in such an environment. As is shown in Svensson and Woodford (2004), even if the monetary authority and the private agents have asymmetric information, certainty equivalence holds in the sense that optimal policy response to estimates of the state of the economy is independent of degree of uncertainty present. However, the separation principle does not hold, since the central bank's estimation is not independent of its chosen policy and the bank's limited information can affect the structure of the equilibrium. In the absence of fiscal foresight, partial information has no effect and the solution paths of endogenous variables are the same as those with full information. However, in the presence of fiscal foresight, the central banker is not able to correctly estimate the exogenous disturbances in the economy. This ultimately causes the monetary authority to induce history dependence in endogenous variables and worsen private agents' welfare.

The identification problem, as noted by Leeper, Walker, and Yang (2008), is due to the fact that when fiscal foresight is present, the solution paths of variables have a VARMA(1, q) representation, where q represents the number of periods of fiscal foresight³. There is no guarantee that a VARMA model has an invertible VAR representation, and as shown by Leeper, Walker, and Yang (2008), with foresight the VARMA model is not invertible. With fiscal foresight, the structural disturbances span a strictly larger space than do the observables, creating an identification problem. Instead of recovering current structural disturbances, the central bank recovers a linear combination of current and past structural disturbances.

This leads to a potential problem for setting monetary policy as the monetary authority responds to statistical innovations that are in fact different from the underlying structural disturbances in

³This is the case when there is a labor/leisure trade-off in the economy. As noted by Leeper, Walker, and Yang (2008), in a RBC model with capital accumulation and no labor choice, the solution paths have a VARMA(1, $q - 1$) representation. Leeper, Walker, and Yang (2008) also note that the form of the VARMA representation depends on how news is modeled.

the economy. This, in turn, causes the equilibrium of such a model to differ from the equilibrium that the monetary authority desires to reach (i.e. the one where the monetary authority responds to the structural disturbances). We quantify how the resulting equilibrium differs and find that when the central banker only has partial information about the economy, the optimal monetary policy can alter the solution paths of endogenous variables and, in general, create history dependence. In addition, the resulting equilibrium reduces the welfare of private agents. Previous research investigating the effects of partial information on optimal policy has found similar results (see Aoki (2003),(2006)). However, our results differ in that it holds even when the central bank has perfect information about a subset of variables in the economy, whereas previous research has focused on cases where the central bank observes a subset of variables with measurement error. We find that foresight alone affects the monetary authority's optimal policy and show how foresight changes some of the properties of optimal policy.

This paper is organized as follows. Section 2 solves for the optimal monetary policy when fiscal foresight is present in the economy and the central bank has complete information about the state of the economy. In addition, it examines the welfare consequences of fiscal foresight and discusses the differences between the monetary authority's response under optimal policy and various Taylor rules. Section 3 sets up and solves a model where the monetary authority has limited information about the state of the economy and uses the Kalman filter to estimate structural disturbances. It then examines the consequences of such a policy. Finally, section 4 concludes.

2 Foresight Implications with Full Information

We begin by discussing the model used for our analysis and investigating the implications of fiscal foresight when the monetary authority has full information about the state of the economy. We assume the monetary authority implements a policy that minimizes the welfare losses of private agents. Specifically, we consider the effects of both discretionary policy and optimal policy from a timeless perspective, as defined in Woodford (1999b).

Our model is a simple version of the dynamic sticky price models which are often found in the recent literature on monetary policy. The structural equations of the economy consist of a log-linearized Phillips curve and a consumption Euler equation given by

$$\hat{\pi}_t = \kappa[\hat{Y}_t + \psi\hat{\tau}_t] + \beta E_t \hat{\pi}_{t+1} + \hat{a}_t \quad (1)$$

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma(\hat{i}_t - E_t \hat{\pi}_{t+1}) \quad (2)$$

where \hat{i}_t , $\hat{\pi}_t$, and \hat{Y}_t are the nominal interest rate, inflation rate, and level of output at time t , respectively. All three are measured in percentage deviations from their steady-state values in a steady state with zero inflation. The parameter $\kappa > 0$ is a measure of the speed of price adjustment,

$\beta \in (0, 1)$ can be interpreted as the discount rate, $\psi > 0$ is a measure of the effect of distortionary taxes, and $\sigma > 0$ can be thought of as the intertemporal elasticity of substitution of expenditure. The tax rate at time t , $\hat{\tau}_t$, enters the Phillip's curve as an exogenous disturbance to production. We abstract from any fiscal financing consequences of modeling such a tax process and assume that lump-sum taxes/transfers exist and adjust each period so that the government's budget constraint is met. We also introduce a second supply shock \hat{a}_t in the Phillips's curve which we will refer to as a shock to technological productivity. Equations (1) and (2), along with a monetary policy reaction function which sets the nominal interest rate, determine the equilibrium paths of inflation, output, and the interest rate.

To introduce tax foresight into the model, we must specify how news about taxes signals changes in future tax rates. In order to illustrate the effects of tax foresight and keep the model as simple as possible, we assume that tax information flows take a particularly simple form: agents at time t receive a signal that tells them exactly what tax rate they will face in period $t+j$. The log-linearized tax rule is then given by

$$\hat{\tau}_t = u_{t-j}^\tau, \quad u^\tau \sim N(0, \sigma_\tau^2) \quad (3)$$

The subscript on u^τ indicates the period in which information about the value of u^τ is received. Thus, j denotes the degree of fiscal foresight present in the model. Note that this tax rule implies that agents and the central bank at time t have perfect knowledge of $\{\hat{\tau}_{t+j}, \hat{\tau}_{t+j-1}, \dots\}$. For concreteness, we assume that \hat{a}_t evolves according to the process

$$\hat{a}_t = u_t^a, \quad u^a \sim N(0, \sigma_a^2) \quad (4)$$

where u_t^a and u_t^τ are independent and serially uncorrelated at all leads and lags.

The welfare loss the monetary authority seeks to minimize is the expected discounted sum of period loss functions

$$L \equiv E \left[\sum_{t=0}^{\infty} \beta^t \left\{ \frac{q_\pi}{2} \hat{\pi}_t^2 + \frac{q_y}{2} \hat{y}_t^2 \right\} \middle| I_t \right] \quad (5)$$

where \hat{y}_t is the welfare relevant output gap, expressing the difference between output and its efficient level. Since there are two cost push shocks in the model, \hat{a}_t and $\hat{\tau}_t$, the output gap can be written as $\hat{y}_t = \hat{Y}_t + \varrho_a \hat{a}_t + \varrho_\tau \hat{\tau}_t$ for some parameters $\varrho_a, \varrho_\tau \in (0, \infty)$ ⁴. $E[\cdot | I_t]$ represents the expectation with respect to the central bank's information set. For the time being, we assume that the central bank and private agents have the same information set and that both have full information about the state of the economy. Benigno and Woodford (2005) show that this loss function is a quadratic approximation to the sum of the expected utility of a representative agent in a Calvo sticky price model. Thus, ranking policies in terms of L is equivalent to ranking policies based on utility maximization. For reference, the underlying New Keynesian model assumed for the loss function's derivation is reproduced in appendix A, and the derivations of the loss function, along with the

⁴Appendix B derives the relationship between ϱ_a and ϱ_τ and the deep parameters of the model.

structural equations (1) and (2), are presented in appendix B. The loss function (5) is minimized subject to equations (1) and (2).

For future reference, note that equations (1) and (2) can be rewritten in terms of the output gap:

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta E_t \hat{\pi}_{t+1} + \nu_a \hat{a}_t + \nu_\tau \hat{\tau}_t \quad (6)$$

$$\hat{y}_t = E_t \hat{y}_{t+1} - \sigma (\hat{i}_t - E_t \hat{\pi}_{t+1}) + r_t^n \quad (7)$$

where

$$\begin{aligned} \nu_a &\equiv 1 - \kappa \varrho_a, & \nu_\tau &\equiv \kappa(\psi - \varrho_\tau) \\ r_t^n &\equiv \varrho_a \hat{a}_t + \varrho_\tau \hat{\tau}_t - E_t(\varrho_a \hat{a}_{t+1} + \varrho_\tau \hat{\tau}_{t+1}) \end{aligned}$$

and, following Woodford (2003), r_t^n is the natural rate of interest.

2.1 Discretionary Policy

In this section we examine optimal discretionary policy under various degrees of fiscal foresight. Under discretion, the central bank has no control over the agent's expectations and takes them as exogenously given. Letting φ be the Lagrange multiplier on equation (1) and λ be the Lagrange multiplier on equation (2), the first order conditions of the central bank's problem in this case are:

$$\begin{aligned} q_\pi \hat{\pi}_t + \varphi_t &= 0 \\ q_y \hat{y}_t + \lambda_t - \kappa \varphi_t &= 0 \\ \sigma \lambda_t &= 0 \end{aligned}$$

These conditions imply that $\lambda_t = 0$ and the constraint from the consumption Euler equation, equation (2), is not binding. From these first order conditions we get an inflation targeting rule:

$$\hat{\pi}_t = \frac{-q_y}{q_\pi \kappa} \hat{y}_t \quad (8)$$

Substituting this equation into the Phillips curve, equation (6), leads to a difference equation that can be solved for the solution path for the output gap. This difference equation can be written as:

$$\left(1 - \left[\frac{1}{\beta} \left(1 + \frac{\kappa^2 q_\pi}{q_y} \right) \right]^{-1} B^{-1} \right) \hat{y}_t = - \left[\frac{1}{\beta} \left(1 + \frac{\kappa^2 q_\pi}{q_y} \right) \right]^{-1} \frac{\kappa q_\pi}{q_y \beta} (\nu_a \hat{a}_t + \nu_\tau \hat{\tau}_t),$$

where B stands for the backshift operator. Assuming $1 < \frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y})$, the solution path for the output gap is given by

$$\hat{y}_t = -E_t \sum_{j=0}^{\infty} \left[\frac{1}{\beta} \left(1 + \frac{\kappa^2 q_\pi}{q_y} \right) \right]^{-(j+1)} \frac{\kappa q_\pi}{q_y \beta} (\nu^a \hat{a}_{t+j} + \nu^\tau \hat{\tau}_{t+j}) \quad (9)$$

Given our assumptions about the stochastic processes $\hat{\tau}_t$ and \hat{a}_t , the solution for the output gap then reduces to

$$\hat{y}_t = \frac{\frac{-\kappa q_\pi}{q_y}}{1 + \frac{\kappa^2 q_\pi}{q_y}} \left[\nu^a \hat{a}_t + \nu^\tau \left(u_{t-j}^\tau + \frac{1}{\frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y})} u_{t-j-1}^\tau + \dots + \frac{1}{\left(\frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y})\right)^j} u_t^\tau \right) \right] \quad (10)$$

The output gap decreases with either news of a tax increase or the realization of a tax increase. Moreover, more recent news is discounted relative to distant news. Although tax rates are discounted in the usual way (this can be seen from equation (9)), tax news is discounted in the opposite manner, because more recent news affects tax rates farther in the future.

Substituting the solution for the output gap into the targeting rule, equation (8), gives the solution for the inflation path:

$$\hat{\pi}_t = \frac{1}{1 + \frac{\kappa^2 q_\pi}{q_y}} \left[\nu^a \hat{a}_t + \nu^\tau \left(u_{t-j}^\tau + \frac{1}{\frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y})} u_{t-j-1}^\tau + \dots + \frac{1}{\left(\frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y})\right)^j} u_t^\tau \right) \right] \quad (11)$$

These solutions can be substituted into the consumption Euler equation, equation (7), to derive the solution for the interest rate. When $j = 1$, i.e. when there is one period of fiscal foresight, the solution path for the interest rate is given by

$$\hat{i}_t = \left(\frac{\kappa q_\pi \nu^a}{q_y + \kappa^2 q_\pi} + \varrho_a \right) \frac{1}{\sigma} \hat{a}_t + \left(\frac{\kappa q_\pi \nu^\tau}{q_y + \kappa^2 q_\pi} + \varrho_\tau \right) \frac{1}{\sigma} u_{t-1}^\tau + \left[\frac{\nu^\tau (q_y - \kappa q_\pi \frac{1}{\sigma})}{q_y + \kappa^2 q_\pi} + \frac{\beta \kappa q_\pi \nu^\tau q_y}{\sigma (q_y + \kappa^2 q_\pi)^2} - \varrho_\tau \frac{1}{\sigma} \right] u_t^\tau \quad (12)$$

The interest rate increases with the realization of a tax increase, but it may increase or decrease with news of a tax increase. The ambiguity is due to the fact that the interest rate responds to current and expected changes in the output gap and inflation rate. With foresight, the monetary authority and private agent expect the output gap and inflation rate to change in later periods with the realization of the tax policy change. Since the expected change in the output gap and inflation rate are in opposite directions, the interest rate can move in either direction, depending on the relative importance the monetary authority places on minimizing inflation variability versus variability to the output gap. For reasonable parameter calibrations, the interest rate decreases with news of a tax increase.

Examining equations (10) and (11) reveals that fiscal foresight increases the variances of endogenous variables (by introducing moving average terms into their solution paths). This result will be crucial for welfare evaluations. Furthermore, the moving average terms, which appear because

of the presence of fiscal foresight, introduce serial correlation in endogenous variables, even in this simple set-up with uncorrelated shocks.

2.2 The ‘Timeless’ Policy

In this section we examine optimal monetary policy from a timeless perspective, as defined in Woodford (1999b), under various degrees of fiscal foresight. In this case the first order conditions from the central bank’s welfare loss minimization problem are

$$\begin{aligned} q_\pi \hat{\pi}_t - \beta^{-1} \sigma \lambda_{t-1} + \varphi_t - \varphi_{t-1} &= 0 \\ q_y \hat{y}_t + \lambda_t - \beta^{-1} \lambda_{t-1} - \kappa \varphi_t &= 0 \\ \sigma \lambda_t &= 0 \end{aligned}$$

These conditions imply that $\lambda_t = 0$ and the following inflation targeting rule:

$$\hat{\pi}_t = \frac{q_y}{q_\pi \kappa} [\hat{y}_{t-1} - \hat{y}_t] \quad (13)$$

Substituting this rule into the Phillips Curve, equation (6), gives a second order difference equation in the output gap

$$E_t \hat{y}_{t+1} - \underbrace{\frac{1}{\beta} \left[(1 + \beta) + \frac{\kappa^2 q_\pi}{q_y} \right]}_{\alpha_1} \hat{y}_t + \underbrace{\frac{1}{\beta}}_{\alpha_2} \hat{y}_{t-1} = \frac{\kappa q_\pi}{q_y \beta} [\nu_a \hat{a}_t + \nu_\tau \hat{\tau}_t] \quad (14)$$

This difference equation can be factored as

$$(\mu_1 - B^{-1})(\mu_2 - B^{-1})\varphi_{t-1} = \frac{\kappa q_\pi}{q_y \beta} [\nu_a \hat{a}_t + \nu_\tau \hat{\tau}_t] \quad (15)$$

where $\alpha_1 = \mu_1 + \mu_2$ and $\alpha_2 = \mu_1 \mu_2$. Let $\mu_2 = \frac{1}{\beta \mu_1}$. $|\mu_1| < 1$ if

$$\frac{q_y}{q_\pi} > \frac{-\kappa^2}{1 + \beta}$$

For reasonable parameter calibrations, q_y and q_π are both positive and this condition is met. Assuming restrictions on the structural parameters satisfy this condition, the difference equation can be solved for the path of the output gap.

$$\hat{y}_t = \mu_1 \hat{y}_{t-1} - \frac{\kappa q_\pi}{q_y} \sum_{j=0}^{\infty} \beta^j \mu_1^{j+1} E_t [\nu_a \hat{a}_{t+j} + \nu_\tau \hat{\tau}_{t+j}] \quad (16)$$

Given our assumptions about the stochastic processes $\hat{\tau}_t$ and \hat{a}_t , the solution for the output gap then reduces to

$$\hat{y}_t = \mu_1 \hat{y}_{t-1} - \frac{q_{\pi\kappa}\mu_1}{q_y} [\nu_a \hat{a}_t + \nu_{\tau} (u_{t-j}^{\tau} + \beta\mu_1 u_{t-j-1}^{\tau} + \dots + (\beta\mu_1)^j u_t^{\tau})] \quad (17)$$

Similarly to the solution under discretionary policy, more recent tax news is discounted relative to older news, and the output gap decreases with either news of a tax increase or the realization of a tax increase. Using this solution in the targeting rule, equation (13), gives the solution for the inflation rate:

$$\hat{\pi}_t = \left(\frac{q_y}{q_{\pi\kappa}} - \mu_1 \right) \hat{y}_{t-1} + \mu_1 [\nu_a \hat{a}_t + \nu_{\tau} (u_{t-j}^{\tau} + \beta\mu_1 u_{t-j-1}^{\tau} + \dots + (\beta\mu_1)^j u_t^{\tau})] \quad (18)$$

As noted in Woodford (1999b), the optimal monetary policy involves history dependence. This happens because the central bank internalizes the effects of its predictable policy on private sector expectations, and this, consequently, affects the current inflation rate and output gap. Substituting the solutions for inflation and the output gap into the consumption Euler equation, equation (7), gives the solution for the interest rate. Again, the interest rate may increase or decrease with news of a tax increase, but for most parameter calibrations it will decrease. Just as with discretionary policy, from equations (17) and (18) it is clear that fiscal foresight increases the variances and serial correlations of endogenous variables.

Equations (8) and (13) specify targeting rules for optimal monetary policy that are robust in the sense of Giannoni and Woodford (2003a,b). In a model where the central bank has no model uncertainty and perfectly observes \hat{y}_t and $\hat{\pi}_t$, fiscal foresight is irrelevant for monetary policy; the monetary authority commits to a rule that is independent of the tax process. Furthermore, even if the monetary authority mistakenly believed agents did not respond to fiscal news, it would not matter for optimal monetary policy as long as \hat{y}_t and $\hat{\pi}_t$ are perfectly observed. This result is not specific to this particular model; Woodford (2003) shows that many model specifications can be written in terms of a targeting or interest rate rule that is independent of the exogenous processes in the economy. As long as the central banker can observe the output gap, he will have no problem implementing the optimal policy. However, if the monetary authority does not observe the output gap, fiscal foresight presents some challenges in implementing monetary policy. These complications are discussed in section 3.

2.3 Welfare Consequences of Foresight

As noted earlier, using the loss function it is possible to rank policies in terms of the implied value of L . As shown in Appendix B, this is equivalent to ranking policies in terms of their implied value of utility for the private agent. Following Woodford (1999a) and Giannoni (2001), we calculate welfare by taking the unconditional expectation of the loss function over all possible histories of

disturbances. To do this, we calculate

$$\begin{aligned}
& E\{E_0(1 - \beta) \sum_{t=0}^{\infty} \beta^t (\frac{q_\pi}{2} \hat{\pi}_t^2 + \frac{q_y}{2} \hat{y}_t^2)\} \\
= & E(1 - \beta) \{ \sum_{t=0}^{\infty} \beta^t (\frac{q_\pi}{2} E_0[\hat{\pi}_t] + \frac{q_y}{2} E_0[\hat{y}_t] + \frac{q_\pi}{2} \text{Var}_0[\hat{\pi}_t] + \frac{q_y}{2} \text{Var}_0[\hat{y}_t]) \}
\end{aligned}$$

This value indicates the loss (expressed as a percentage of steady state consumption) due to temporary disturbances in excess of the steady-state loss. Variances were computed using the doubling algorithm described in Hansen and Sargent (1997). Parameter calibrations are reported in Table 1. These values are taken from Benigno and Woodford (2003) and are standard values used in the literature⁵.

Table 2 displays the results. Using an unconditional welfare measure, it appears that foresight does not drastically alter the welfare losses in an economy. It also appears that foresight cannot improve welfare. However, this result is not surprising. Foresight does not change the unconditional means of the endogenous variables; the means are zero with or without fiscal news. Foresight does increase the variability of endogenous variables⁶, which causes the small reduction in the unconditional welfare of private agents.

These results are not specific to the benchmark calibrations. Figure 6 displays unconditional welfare losses for discretionary policy for various values of κ (first panel) and σ (second panel) and under various degrees of fiscal foresight. All other parameter values are kept at benchmark values. The results show that foresight does not improve welfare unconditionally. Additional sensitivity analysis looking at other parameter values and standard deviation values have found the same result⁷. Interestingly, this result is not specific to tax news. In this simple model, taxes are a form of a cost push shock, and any anticipated change to cost push disturbances will have the same result. For instance, Winkler and Wohltmann (2008) find a similar result for anticipated oil price shocks in a similar model.

One problem with the unconditional welfare measure is that it misses an important aspect of fiscal news: *conditional* on the history of disturbances, foresight gives agents more information about the expected value of future endogenous variables. By integrating over the history of all possible disturbances, the unconditional measure negates this effect. Thus, by taking into account the informational gain in news, the conditional welfare rankings can differ substantially from the unconditional rankings.

⁵Sensitivity analysis showed that the welfare rankings do not depend on the relative standard deviations of the shocks or the other parameter calibrations. These additional results are available from the author.

⁶This is clear from calculating the variances of inflation and the output gap from equations (10)-(11) or equations (17)-(18).

⁷These results are available from the author upon request.

Table 3 demonstrates this result with loss values at time t conditional on variables being at their steady state values until time $t - 1$, when positive or negative tax news (one standard deviation from the mean) hit the economy. News of an increase in the tax rate increases expectations of future inflation while reducing expectations of future output gaps. Because the welfare measure, equation (5), rates changes in inflation more heavily (that is, $q_\pi > q_y$ for reasonable parameter calibrations), this effect translates into increases in the conditional loss values that increase as the degree of foresight multiplies. In contrast, news of a decrease in taxes reduces expectations of future inflation and decreases the conditional loss values. Thus, it appears good news improves the private agents' welfare, while bad news reduces it. Because the welfare measure, equation (5), rates changes in inflation more heavily, these expected reductions in inflation can dramatically decrease the welfare losses. Note that if welfare is measured conditional on all variables being at steady state levels in the past, the welfare rankings are the same as the unconditional rankings (since in this case, in periods $\tau < t$ agents do not receive any news about the path of future variables).

2.4 Comparisons with Alternative Policies

As suggested above, for reasonable calibrations of parameters, the interest rate always decreases with news of a tax increase. In stark contrast, if the monetary authority follows a simple Taylor rule where the interest rate responds to inflation and output, then the interest rate will usually increase with news of a tax increase⁸. Even if the Taylor rule is a function of output growth (instead of output), the interest rate response usually will be positive⁹.

Figure 1 compares impulse responses for the model economy with various monetary policies and no fiscal foresight. The interest rate response to an unanticipated tax increase is qualitatively the same for all the monetary policies considered. However, the response under the Taylor rules is quantitatively smaller; the Taylor rule places more attention on interest rate stability than the optimal policy calls for. Figures 2-3 compare impulse responses for the model economy with alternative monetary policies and varying degrees of fiscal foresight. With foresight, the interest rate responses are qualitatively different from the optimal policy responses. These rules involve the monetary authority responding to fiscal news by moving the interest rate in the *opposite* direction from the response under the optimal monetary policy.

Remarkably, this result has no consequential welfare implications. Table 2 displays welfare calculations (see section 2.3 for an explanation of how these results were calculated) under alternative policies. In this model, even without foresight the Taylor rule is not a good approximation of the optimal policy, and the increased losses from fiscal foresight are not severe. Interestingly, both unconditionally and conditionally welfare increases as the degree of fiscal foresight increases.

⁸See Appendix C for analytical solutions to the model when the monetary authority follows a simple Taylor rule.

⁹Analytical solutions are not available in this case. This result is based on numerical simulations for various calibrations of parameters.

3 Foresight Implications with Imperfect Information

It is not obvious how the monetary authority can implement the optimal policies described in the previous section. In a real economy, it is almost impossible for the central bank to directly observe the supply shocks u^τ and u^s . Instead, the monetary authority must infer these values from observable variables such as the inflation and tax rates. This complicates matters for the central bank in that the monetary authority must estimate these disturbances in order to implement its monetary policy. To explore the repercussions of this issue in the presence of fiscal foresight, we assume now that the monetary authority does not observe the path of all structural disturbances but does perfectly observe a subset of all current and past variables. In order to illustrate the effects of tax foresight and keep the model as simple as possible, we assume that there is one period of tax news, that is $\hat{\tau}_t = u_{t-1}^\tau$.

To focus the analysis on the implications of the monetary authority's limited information set, we abstract from any uncertainty facing the private agents. We assume private agents have full information about the current state and history of the economy, including the realizations of the exogenous disturbances. One justification for this assumption is that private agents, as workers in firms, have more information about variations in production capacity (due to technological productivity shocks) than the monetary authority does and that private agents, when given tax news, are able to calculate by how much their personal, future tax rates will change and plan their personal consumption/savings response to such changes. We define $E_t z_\tau$ to be the rational expectation of the variable z_τ given the private agent's information in period t . We define $z_{\tau|t}$ to be the rational expectation of variable z_τ given the monetary authority's information in period t .

We assume the following time sequence. At the beginning of time t , disturbances u_t^τ and u_t^s hit the economy. Then, all endogenous variables are determined simultaneously. Based on the observation of its subset of variables, the central bank forms its estimates about $\{u_t^\tau, u_t^s\}$, and sets its policy instrument in order to minimize the loss function (5). We assume that the central bank uses the Kalman filter to form its estimates of the current disturbances. In the absence of fiscal foresight, the solution paths of endogenous variables are the same as those with full information¹⁰. However, in the presence of fiscal foresight, the central banker is not able to correctly estimate the exogenous disturbances in the economy and can induce history dependence in the solution paths of endogenous variables. This happens because the solution paths of the observables have a VARMA representation with the presence of fiscal foresight, and the VARMA does not have an invertible VAR representation. Thus, the equilibrium moving average representation of the path of the observable variables is nonfundamental, i.e. the structural disturbances cannot be recovered from current and past observables; the current and past structural disturbances span a strictly larger space than the observables in this case.

¹⁰This follows from our assumptions that the central bank perfectly observes a subset of variables and knows the underlying structural parameters of the economy.

The central banker does not observe the nonfundamental moving average representation of the economy; instead, he recovers the observationally equivalent Wold representation. It turns out that the statistical innovations of this representation are weighted averages of current *and* past structural disturbances. It is these statistical innovations that the monetary authority recovers from the Kalman filter. Thus, the central banker ends up responding to innovations that are different from the underlying structural disturbances in the economy; instead, he responds to weighted averages of current and past structural disturbances.

As demonstrated in Svensson and Woodford (2004), since the loss function is quadratic and the structural equations are linear, certainty equivalence holds given our assumptions about the private agent's and central bank's information sets. That is, the optimal policy is the same as if the state of the economy were fully observable, except that the central bank responds to an efficient estimate of the state of the economy rather than to its actual value. We assume that the central bank would like to achieve the discretionary optimal policy by employing the following inflation targeting rule

$$\hat{\pi}_{t|t} = \frac{-q_y}{q_\pi \kappa} \hat{y}_{t|t} \quad (19)$$

In order to follow such a rule, the monetary authority sets the interest rate according to

$$\hat{i}_t = \underbrace{\left(\frac{\kappa q_\pi \nu^a}{q_y + \kappa^2 q_\pi} + \varrho_a \right) \frac{1}{\sigma} u_{t|t}^a}_{f_1} + \underbrace{\left(\frac{\kappa q_\pi \nu^\tau}{q_y + \kappa^2 q_\pi} + \varrho_\tau \right) \frac{1}{\sigma} u_{t-1|t}^\tau}_{f_2} + \underbrace{\left[\frac{\nu^\tau (q_y - \kappa q_\pi \frac{1}{\sigma})}{q_y + \kappa^2 q_\pi} + \frac{\beta \kappa q_\pi \nu^\tau q_y}{\sigma (q_y + \kappa^2 q_\pi)^2} - \varrho_\tau \frac{1}{\sigma} \right] u_{t|t}^\tau}_{f_3} \quad (20)$$

This model falls within the general framework set out in Svensson and Woodford (2004) to solve for the optimal policy under asymmetric information. Thus, we proceed to solve this model by first transforming the model into Svensson and Woodford's notation and then use their procedure to derive the solution. Towards this end, we define the vector X_t of predetermined variables and the vector x_t of forward-looking endogenous variables as

$$X_t \equiv \begin{bmatrix} u_t^a \\ u_t^\tau \\ u_{t-1}^\tau \end{bmatrix}, \quad x_t \equiv \begin{bmatrix} \hat{\pi}_t \\ \hat{Y}_t \end{bmatrix}$$

We can then write the model, conditional on the central bank's information set, as

$$\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = \begin{bmatrix} u_{t+1}^a \\ u_{t+1}^\tau \\ u_t^\tau \\ \hat{\pi}_{t+1|t} \\ \hat{Y}_{t+1|t} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{\beta} & 0 & -\frac{\kappa \psi}{\beta} & \frac{1}{\beta} & \frac{-\kappa}{\beta} \\ \frac{\sigma}{\beta} & 0 & \frac{\sigma \kappa \psi}{\beta} & \frac{-\sigma}{\beta} & 1 + \frac{\sigma \kappa}{\beta} \end{bmatrix}}_A \begin{bmatrix} u_t^a \\ u_t^\tau \\ u_{t-1}^\tau \\ \hat{\pi}_t \\ \hat{Y}_t \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \sigma \end{bmatrix}}_B \hat{i}_t + \begin{bmatrix} u_{t+1}^a \\ u_{t+1}^\tau \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (21)$$

All that remains is to use the Kalman filter to determine the monetary authority's estimates of

predetermined variables $X_{t|t}$. Given these estimates, the central bank sets the interest rate according to equation (20). However, when the monetary authority's observables include endogenous variables, the central bank must sort through a simultaneity problem when employing the Kalman filter. Observed inflation and output are forward-looking variables that depend on both the agent's current expectations of future inflation and output and the current monetary policy. The central bank's current expectations and monetary policy in turn depend on the estimates of the structural disturbances. These in turn depend on the observations of inflation and/or output, completing the circle. In the next sections, we use the method of Svensson and Woodford (2004) to solve for the equilibrium in such a framework.

3.1 Tax and Inflation Observables

In this section we assume that the monetary authority observes the sequence of current and past inflation and tax rates, $\{\hat{\tau}_{t-j}, \hat{\pi}_{t-j}\}_{j=0}^{\infty}$. The monetary authority uses these observables in the Kalman filter to recover the structural disturbances.

In this case, in equilibrium the forward-looking variables, $\hat{\pi}_t$ and \hat{Y}_t , will depend on both the true values of inflation and output and the monetary authority's estimates of the predetermined variables in the economy $X_{t|t}$, where the relationship can be written as

$$\begin{bmatrix} \hat{\pi}_t \\ \hat{Y}_t \end{bmatrix} = \begin{bmatrix} 1 & \kappa\psi(\beta + \kappa\sigma) - f_2\sigma\kappa(1 + \beta + \kappa\sigma) & \kappa\psi \\ 0 & -\sigma[f_2(1 + \kappa\sigma) - \kappa\psi] & 0 \end{bmatrix} \begin{bmatrix} u_t^a \\ u_t^\tau \\ u_{t-1}^\tau \end{bmatrix} + \begin{bmatrix} -f_1\kappa\sigma & -f_3\kappa\sigma & -f_2\kappa\sigma \\ -f_1\sigma & -f_3\sigma & -f_2\sigma \end{bmatrix} \begin{bmatrix} u_{t|t}^a \\ u_{t|t}^\tau \\ u_{t-1|t}^\tau \end{bmatrix} \quad (22)$$

Derivations of equation (22) are given in Appendix D. If the monetary authority correctly estimates the structural disturbances, that is $X_{t|t} = X_t$, equation (22) reduces to the solution paths of inflation and output under full information (which can be found from equations (10) and (11)). However, given the monetary authority's observables, its estimates at time t of the predetermined variables X_t are given by

$$X_{t|t} = \begin{bmatrix} \frac{\kappa\sigma_a^2(\psi - f_2\sigma)}{\phi} & \frac{-\sigma_a^2}{\phi} \\ \frac{\kappa^2\sigma_\tau^2(f_2\sigma - \psi)[f_2\sigma(1 + \beta + \kappa\sigma) - \psi(\kappa\sigma + \beta)]}{\phi} & \frac{\kappa\sigma_\tau^2[f_2\sigma(1 + \beta + \kappa\sigma) - \psi(\beta + \kappa\sigma)]}{\phi} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{\tau}_t \\ \hat{\pi}_t \end{bmatrix} \quad (23)$$

Substituting this expression into equation (22), we can solve for the solution paths of inflation and output in terms of the predetermined variables. In this case, the solutions differ from those with

full information and are of the form

$$\hat{\pi}_t = \phi_1 u_t^a + \phi_2 u_t^\tau + \frac{\nu_\tau}{1 + \frac{\kappa^2 q_\pi}{q_y}} u_{t-1}^\tau \quad (24)$$

$$\hat{Y}_t = \phi_3 u_t^a + \phi_4 u_t^\tau - \frac{\varrho_\tau q_y + q_\pi \kappa^2 \psi}{q_y + q_\pi \kappa^2} u_{t-1}^\tau \quad (25)$$

Derivations and expressions for the ϕ_i 's are given in Appendix D. Comparing these expressions to equations (10) and (11) shows that the responses of endogenous variables to the realization of a tax change (u_{t-1}^τ) are the same as in the full information case. This stems from the fact that the monetary authority directly observes the realization of tax changes at time t by observing $\hat{\pi}_t$. However, because the observables do not span the same space as the structural disturbances, the central bank does not recover u_t^a and u_t^τ from the Kalman filter, and the responses of endogenous variables to changes in technological productivity or tax news differ from the full information case.

As long as the tax process is exogenous and taxes are an observable, this result will hold for alternative specifications of the tax and technology process as well. For instance, it holds if taxes or technology followed $AR(p)$ processes. As long as taxes are an observable, the monetary authority can recover the realization of tax changes at time t (u_{t-1}^τ) and will always respond to the realization in the same manner as in the full information case. However, as will be discussed in the next section, once taxes are no longer an observable, the central bank is no longer able to correctly identify the history of tax changes, and its estimates induce history dependence in the solution paths of forward-looking variables.

3.2 Inflation and Technological Productivity Observables

In this section we assume that the monetary authority observes the sequence of current and past inflation and technological progress, $\{\hat{a}_{t-j}, \hat{\pi}_{t-j}\}_{j=0}^\infty$. The monetary authority uses these observables in the Kalman filter to recover the structural disturbances.

In this case, the monetary authority's estimates of the predetermined variables X_t are of the form

$$\begin{bmatrix} u_{t|t}^a \\ u_{t|t}^\tau \\ u_{t-1|t}^\tau \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_1 & 0 \\ 0 & c_2 & 0 \end{bmatrix} \begin{bmatrix} u_{t-1|t-1}^a \\ u_{t-1|t-1}^\tau \\ u_{t-2|t-1}^\tau \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ c_3 & c_4 \\ c_5 & c_6 \end{bmatrix} \begin{bmatrix} u_t^a \\ \hat{\pi}_t \end{bmatrix} \quad (26)$$

for some constant c_i 's satisfying $|c_1| < 1$, $|c_2| < 1$ ¹¹. Notice that this implies the central bank's

¹¹Analytical solutions are not available in this case. These results are based on numerical results for various parameter calibrations.

estimate of the tax news, $u_{t|t}^\tau$, depends on the entire history of past observables:

$$u_{t|t}^\tau = \frac{1}{1 - c_1 L} (c_3 u_t^a + c_4 \hat{\pi}_t)$$

Thus, instead of recovering the correct estimate of the tax news, the monetary authority instead recovers a weighted average of all current and past structural disturbances. This, in turn, causes the solution paths of endogenous variables to exhibit history dependence. To see this, note that substituting equation (26) into equation (20) implies that the interest rate is set according to the rule

$$\hat{i}_t = c_1 \hat{i}_{t-1} + (f_1 + f_2 c_5)(1 - c_1 L) u_t^a + (f_2 c_2 + f_3) c_3 u_t^a + f_2 c_6 (1 - c_1 L) \hat{\pi}_t + (f_2 c_2 + c_3) c_4 \hat{\pi}_t \quad (27)$$

Interestingly, it follows from equation (27) that the monetary authority's policy involves the solution path of the interest rate responding to the lagged interest rate. The degree of interest-rate smoothing depends on the value of c_1 , the parameter governing how much weight the central bank places on its previous estimate of tax news when updating its estimate of such news. It has been mentioned by many that the federal funds rate is serially correlated¹². Looking at the solution path for the interest rate, it is clear that the interest rate will be serially correlated in this set-up. This is due to the presence of tax foresight and partial information. Equation (27) also shows that the interest rate responds to last period's inflation but does not involve lagged variables beyond time $t - 1$.

The interest rate is not the only variable that exhibits history dependence. The solution paths for inflation and output are of the form:

$$\hat{\pi}_t = \zeta_1^\pi \hat{\pi}_{t-1} + (\zeta_2^\pi - \zeta_1^\pi \zeta_3^\pi L) u_t^a + (\zeta_4^\pi - \zeta_1^\pi \zeta_5^\pi L - \zeta_1^\pi \zeta_6^\pi L^2) u_t^\tau \quad (28)$$

$$\hat{Y}_t = \zeta_1^y \hat{Y}_{t-1} + (\zeta_2^y - \zeta_1^y \zeta_3^y L) u_t^a + (\zeta_4^y - \zeta_1^y \zeta_5^y L - \zeta_1^y \zeta_6^y L^2) u_t^\tau \quad (29)$$

for some constant ζ_i^j 's. The history dependence in the solution paths is a direct consequence of the presence of fiscal foresight and the limited information available to the central bank. Interestingly, even in this simple set-up where the structural disturbances are uncorrelated, the solution paths of endogenous variables involve lagged endogenous variables. Previous research investigating the consequences of the central bank having limited information (see Aoki (2003),(2006)) has focused on circumstances where the monetary authority observes a noisy measurement of current variables¹³. In such cases, the monetary authority's policy can induce history dependence in the solution path of variables if the structural shocks follow AR(p) processes. Our analysis shows that alternatively the central bank's policy can induce history dependence if some structural disturbances have MA(q) components. The reason why the solution paths of endogenous variables involve lagged variables

¹²Examples include Rudebusch (1995) and Goodfriend (1991).

¹³In models without foresight, it is necessary to introduce such noise to make the partial information case nontrivial.

is due to the fact that inflation and technological productivity are not sufficient statistics for the identification of the underlying structural disturbances.

It is important to note that as long as the central bank does not include taxes in its observables, the solution paths of endogenous variables will exhibit history dependence, no matter what observables the monetary authority uses. Without taxes as an observable, the monetary authority cannot recover the history of past tax disturbances, and, in turn, the estimate of the tax news will be a weighted average of all current and past structural disturbances. It is also interesting to note that unless the monetary authority directly observes a particular structural disturbance, fiscal foresight's presence will make it impossible to correctly estimate the specific disturbance.

3.3 Consequences of Partial Information

As previously noted, when the central bank has partial information about the current state of the economy, it can change the structure of the equilibrium and induce history dependence. As discussed below, this affects the economy's impulse responses following structural disturbances. In addition, the bank's policy affects what information about the structural disturbances the monetary authority is able to recover and can have detrimental welfare implications.

Figures 4 and 5 give impulse response functions for news of a 1% tax increase and 1% decrease in technological productivity respectively in the model economy when the central bank has access to various observables. The results are compared to the result under discretionary policy with full information, the policy the monetary authority would like to achieve.

When taxes are an observable, the monetary authority correctly estimates the history of tax disturbances and only has trouble identifying the current tax disturbance. Because the central bank can identify past tax disturbances, the impulse response following a 1% tax increase only differs from the response under full information in the initial period, when the central bank's estimate of the disturbance is inaccurate. Although it appears the impulse response functions for a technology shock are the same quantitatively for the different policies (see Figure 5), the responses do vary slightly, although the differences are quantitatively trivial.

When taxes are not included in the observables, the response following a tax disturbance changes qualitatively. This is because the central bank's policy creates history dependence in the solution paths of variables (this can be seen by comparing equations (11) and (28)) and it takes the economy longer to return to its original steady state following tax news. The response following an unanticipated technology shock does not change quantitatively (since technological productivity is an observable in this case). It is important to note that even in more complex models where the monetary authority might not observe technological productivity, the responses following a technology shock would probably not differ qualitatively from the response under full information. This

is because of our assumption that there is no news about technological productivity, which implies that the statistical innovations that the monetary authority observes will only involve the current technology shock, as opposed to a linear combination of current and past technology shocks (as happens with the anticipated tax change).

Interestingly, for both sets of observable variables, the interest rate increases with news of a tax increase, while the interest rate decreases when the central bank has full information. This is partly due to the central bank's inaccurate estimate of the tax disturbance: the monetary authority mistakenly estimates the disturbance to be less severe than it is and responds, in turn, to this smaller disturbance. This result is also partly due to the central bank acting cautiously under partial information and, when inflation and technological productivity are observables, wanting to smooth the path of the interest rate. When the central bank estimates the tax news to be positive, it expects that it will have to increase interest rates in the next period with the realization of the tax change. This makes the monetary authority less willing to decrease the interest rate in the current period.

Table 4 displays welfare calculations (see section 2.3 for an explanation of how these results were calculated) under policies where the central bank has various observables. The central bank's policy always reduces welfare both unconditionally and conditionally (compared to the full information policy). Interestingly, when taxes are included in the observables, welfare losses are close to the losses under full information. This is because the solution paths of endogenous variables are almost the same as the paths under the full information policy (which can be seen by comparing equations (11) and (24)). This result is due to the fact that taxes are completely exogenous in this set-up. If the tax process responded to endogenous variables (such as output or government debt), the monetary authority would not be able to correctly estimate the previous period's tax rate, and its estimate of the tax news would depend on the entire history of past tax changes.

Although the above results were for specific, simple examples, several results generalize. When taxes are exogenous, identification mistakes will be minimized when taxes are included in the observables. In general, the central bank's policy induces history dependence in the model and increases the variances and serial correlations of endogenous variables. Unless the monetary authority directly observes a particular structural disturbance, fiscal foresight's presence will make it impossible to correctly estimate the particular disturbance and thus, inferences from the central bank's estimates of structural disturbances will be misleading. Qualitatively, these results hold for longer periods of fiscal foresight as well. It is important to note that our examples were for the simplest and *best-case* scenarios. With a more complicated model or more complicated stochastic processes for the structural disturbances, the innovations the monetary authority recovers will differ more from the true structural disturbances, causing more history dependence in the solution paths of variables and worsening welfare.

4 Conclusion

This paper investigated the theoretical consequences of fiscal foresight on monetary policy. When the central bank can observe the current output gap, foresight does not affect the optimal policy. However, the resulting optimal response to fiscal news can be qualitatively different from the response under alternative monetary policy rules. In addition, foresight can be welfare improving or worsening, depending on the welfare measure and on monetary policy.

When the central bank does not directly observe the current output gap and must recover it from estimates of the structural disturbances, the central bank's policy can actually change the structure of the equilibrium. This is due to the fact that the observables do not span the same space as the structural disturbances, which causes an identification problem and leads to a difference between the statistical innovations that the monetary authority recovers and the underlying structural disturbances in the economy. In this case the central bank can induce history dependence in the solution paths of endogenous variables and reduce the welfare of the economy.

There are two directions in which we would like to expand this research. The first is to give a more thorough analysis of the consequences of fiscal foresight under partial information. Towards this end, we are currently investigating the effects of fiscal foresight under alternative specifications about news information flows and alternative estimation techniques the monetary authority can employ¹⁴. In addition, we hope to extend the analysis and consider the effects of fiscal foresight when the central bank has limited information and would like to implement the optimal policy under commitment.

In addition, it would be interesting to relax the information set of agents (to a more realistic assumption) and consider how the results change. For this case one could assume that agents do not know how the central bank recovers structural disturbances and must form expectations of the central bank's expectations. Thus, higher order expectations would need to be modeled.

¹⁴Because the current and past structural disturbances span a strictly larger space than the observables, no matter what method the monetary authority employs to estimate the disturbances, he will face the same identification issue and not be able to correctly identify the structural disturbances from the observables.

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A The Model

This appendix describes the theoretical model. The model is a version of the Benigno and Woodford (2005) model, extended to allow advance news of tax policy changes. The economy consists of a representative household, a representative final goods producing firm, a continuum of intermediate goods producing firms (each indexed by $i \in [0, 1]$), a monetary authority, and the government. Each intermediate good is supplied by a monopolistically competitive producer. We assume there are an infinite number of industries (each indexed by $j \in [0, 1]$), producing many differentiated intermediate goods, and that labor is specific to an industry.

A.1 Households

A representative household seeks to maximize the expected utility of its consumption and leisure:

$$U \equiv E_t \sum_{t=0}^{\infty} \beta^t [u(C_t) - \int_0^1 v(H_t(j)) dj] \quad (\text{A.1})$$

where $\beta \in (0, 1)$, $H_t(j)$ is the amount of labor of type j supplied, and C_t is the aggregate consumption of a continuum of differentiated goods which, following Dixit and Stiglitz (1977), is defined as

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad (\text{A.2})$$

where $\theta > 1$ is the elasticity of substitution. Following Benigno and Woodford (2005), the following functional forms are assumed for the utility function:

$$u(C_t) \equiv \frac{C_t^{1-\tilde{\sigma}}}{1-\tilde{\sigma}} \quad (\text{A.3})$$

$$v(H_t) \equiv \frac{\lambda}{1+\nu} H_t^{1+\nu} \quad (\text{A.4})$$

where $\tilde{\sigma}, \nu > 0$. The household's nominal flow budget constraint is

$$P_t C_t + E_t[Q_{t,t+1} A_{t+1}] = A_t + P_t \int_0^1 w_t(j) H_t(j) dj - T_t \quad (\text{A.5})$$

where $w_t(j)$ is the real wage in industry j , T_t is a lump sum transfer, P_t is the price index, and $Q_{t,t+1}$ is a stochastic discount factor such that the price of any bond portfolio at period t with the random value A_{t+1} in period $t+1$ is

$$W_t = E_t[Q_{t,t+1} A_{t+1}] \quad (\text{A.6})$$

where W_t is the household's end-of-period bond portfolio. The household's first order conditions from maximizing its expected utility, equation (A.1), subject to its budget constraint, equation (A.5), give

$$Q_{t,t+1} = \frac{\beta u_c(C_{t+1})}{u_c(C_t)} \frac{P_t}{P_{t+1}} \quad (\text{A.7})$$

$$w_t(j) = \frac{v_h(H_t(j))}{u_c(C_t)} \quad (\text{A.8})$$

A.2 The Final Good Producer

A representative final goods producer uses $y_t(i)$ units of each intermediate good i to produce the final goods, Y_t , according to the constant returns to scale technology due to Dixit and Stiglitz (1977)

$$\left[\int_0^1 y_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \geq Y_t \quad (\text{A.9})$$

Denote the price of the intermediate good as $p_t(i)$. Then the final goods firm's problem is to choose Y_t and $y_t(i)$ to maximize

$$P_t Y_t - \int_0^1 p_t(i) y_t(i) di \quad (\text{A.10})$$

subject to equation (A.9). The first order conditions give

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta} \quad (\text{A.11})$$

Substituting equation (A.11) into equation (A.9) leads to an expression for P_t that must hold at equilibrium

$$P_t = \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{1/(1-\theta)} \quad (\text{A.12})$$

The aggregate resource constraint for the economy is given by

$$Y_t = C_t \quad (\text{A.13})$$

A.3 Intermediate Goods Producers

The intermediate goods producers are monopolistic competitors in their product market and take factor prices as given. Given the price $p_t(i)$ that a firm charges for its product, it is assumed the firm must produce enough to meet the demand for its good. I assume a common technology for the production of all goods. Specifically, firm i hires $h_t(i)$ units of labor to produce $y_t(i)$ according to the technology

$$y_t(i) = A_t f(h_t(i)) = A_t h_t(i)^{1/\phi} \quad (\text{A.14})$$

where $\phi > 1$ and $\{A_t\}$ is a bounded, exogenous process for technological productivity. Following Calvo (1983), producers in each industry fix the prices of their goods for a random interval of time. Each period, $0 \leq \alpha < 1$ fraction of producers in an industry are unable to change their price. The $1 - \alpha$ fraction of suppliers who do change their price in period t choose a new price, $p_t(i)$, that maximizes the expected sum of future profits

$$E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi(p_t(i), p_T^j, P_T; Y_T, \xi_T) \right\} \quad (\text{A.15})$$

where ξ_T is a vector of the realization of exogenous variables at time T . The profit function is defined as

$$\begin{aligned} \Pi(p_t(i), p_T^j, P_T; Y_T, \xi_T) \equiv \\ (1 - \tau_t) p_t Y_t \left(\frac{p_t}{P_t} \right)^{-\theta} - \frac{v_h \left(f^{-1} \left(\frac{Y_t}{A_t} \left(\frac{p_t^j}{P_t} \right)^{-\theta} \right) \right)}{u_c(Y_t)} P_t f^{-1} \left(\frac{Y_t}{A_t} \left(\frac{p_t^j}{P_t} \right)^{-\theta} \right) \end{aligned} \quad (\text{A.16})$$

where $\tau_t \in [0, 1]$ is a tax on nominal profits. Note that I have substituted the industry wage, $w_t(j)$, into equation (A.16) using equation (A.8).

Assuming a symmetric equilibrium where $p_t = p_t^j = p_t^*$, the solution to equation (A.15) is

$$E_t \left\{ \prod_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[(1 - \tau_T)(1 - \theta) Y_T \left(\frac{p_t^*}{P_T} \right)^{-\theta} + \theta \frac{v_h}{u_c} \frac{Y_T}{A_T} \left(\frac{p_t^*}{P_T} \right)^{-\theta-1} \frac{1}{f' f^{-1} \left(\frac{Y_t}{A_t} \left(\frac{p_t^*}{P_t} \right)^{-\theta} \right)} \right] \right\} \quad (\text{A.17})$$

Substituting for $Q_{t,T}$ from equation (A.7) gives

$$\begin{aligned} & E_t \left\{ \prod_{T=t}^{\infty} (\alpha\beta)^{T-t} u_c(Y_T) (1 - \tau_T) Y_T \left(\frac{P_T}{P_t} \right)^{\theta-1} \left(\frac{p_t^*}{P_t} \right)^{-\theta} \right\} \\ & = E_t \left\{ \prod_{T=t}^{\infty} (\alpha\beta)^{T-t} v_y(Y_T) Y_T \frac{\theta}{\theta-1} P_T^\theta P_t p_t^* \frac{1}{A_T f' f^{-1} \left(\frac{Y_t}{A_t} \left(\frac{p_t^*}{P_t} \right)^{-\theta} \right)} \right\} \end{aligned} \quad (\text{A.18})$$

Equation (A.18) can be written compactly as

$$\left(\frac{p_t^*}{P_t} \right)^{1+\omega\theta} = \frac{K_t}{F_t} \quad (\text{A.19})$$

where

$$K_t \equiv E_t \left\{ \prod_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\theta}{\theta-1} v_y(Y_T) Y_T \left(\frac{P_T}{P_t} \right)^{\theta(1+\omega)} \right\} \quad (\text{A.20})$$

$$F_t \equiv E_t \left\{ \prod_{T=t}^{\infty} (\alpha\beta)^{T-t} (1 - \tau_T) Y_T u_y(Y_T) \left(\frac{P_T}{P_t} \right)^{\theta-1} \right\} \quad (\text{A.21})$$

and $\omega = \phi(1 + \nu) - 1 > 0$. With Calvo pricing, the price index evolves according to

$$P_t = [(1 - \alpha)p_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}]^{1/1-\theta} \quad (\text{A.22})$$

Combining equation (A.19) with the price index gives the aggregate supply equation

$$\frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} = \left(\frac{K_t}{F_t} \right)^{\frac{1-\theta}{1+\omega\theta}} \quad (\text{A.23})$$

where $\Pi_t \equiv P_t/P_{t-1}$.

A.4 Monetary and Fiscal Policy

We assume the central bank has control of the riskless short-term nominal interest rate, i_t , which can be defined as

$$1 + i_t = [E_t Q_{t,t+1}]^{-1} \quad (\text{A.24})$$

We assume the monetary authority sets policy by minimizing the welfare losses from the welfare-theoretic loss function (details of the monetary authority's problem are given in the text, and details of the derivations of the welfare-theoretic loss function are given in Appendix B).

We assume lump-sum taxes/transfers adjust each period so that the government budget constraint is met.

Thus, we abstract from the fiscal consequences of fiscal foresight and alternative monetary policies.

B Derivations of Equations in the Paper

This Appendix gives derivations of the equations in the text. Many of these derivations closely follow those of Benigno and Woodford (2005).

B.1 Derivation of the Consumption Euler Equation, (2)

Substituting the aggregate resource constraint, (A.13), and equation (A.24) into equation (A.7) and log linearizing yields

$$\hat{i}_t = \sigma^{-1}[E_t \hat{Y}_{t+1} - \hat{Y}_t] - E_t \hat{\pi}_{t+1}$$

where $\sigma^{-1} \equiv \bar{\sigma}^{-1} \frac{Y}{C} > 0$. Rearranging the equation yields

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma^{-1}(\hat{i}_t - E_t \hat{\pi}_{t+1})$$

This equals equation (2) in the text.

B.2 Second Order Approximation of AS Equation, (A.23)

We derive a second order approximation of the Phillips Curve equation, (A.23), since this will be needed to derive the quadratic welfare measure. Reducing this result to a first order approximation gives equation (1) in the text. From equation (A.23),

$$\log \left(\frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} \right) = \frac{\theta - 1}{1 + \omega\theta} [\log F_t - \log K_t] \quad (\text{B.1})$$

A second order Taylor series expansion of the left-hand side with respect to Π_t around $\bar{\Pi} = 1$ yields

$$\begin{aligned} \log \left(1 - \frac{\alpha \Pi_t^{\theta-1}}{1 - \alpha} \right) &= -\frac{\alpha(\theta - 1)}{1 - \alpha} \left[\hat{\pi}_t + \left(1 + \frac{\theta - 2 + \alpha}{1 - \alpha} \right) \frac{1}{2} \hat{\pi}_t^2 \right] \\ &= -\frac{\alpha(\theta - 1)}{1 - \alpha} \left[\hat{\pi}_t + \frac{1}{2} \frac{(\theta - 1)}{1 - \alpha} \hat{\pi}_t^2 + \mathcal{O}(\|\xi\|^3) \right] \end{aligned} \quad (\text{B.2})$$

where, following Benigno and Woodford (2005), $\mathcal{O}(\|\xi\|^3)$ will be used throughout as shorthand for $\mathcal{O}(\|\xi, \hat{\Delta}_{t_0-1}^{1/2}, \hat{X}_{t_0}\|^3)$ where \hat{X}_{t_0} are state-contingent commitments for period t_0 . Now note that $\log \bar{F} = \log \bar{K}$. Then equation (B.1) can be written as

$$\hat{\pi}_t + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_t^2 = \frac{1 - \alpha}{\alpha(1 + \omega\theta)} (\hat{K}_t - \hat{F}_t) + \mathcal{O}(\|\xi\|^3) \quad (\text{B.3})$$

We now proceed to derive a second order approximation of the right-hand side of equation (B.1). Note that

$$\hat{K}_t + \frac{1}{2} \hat{K}_t^2 + \mathcal{O}(\|\xi\|^3) = (1 - \alpha\beta) E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\hat{k}_{t,T} + \frac{1}{2} \hat{k}_{t,T}^2] + \mathcal{O}(\|\xi\|^3) \quad (\text{B.4})$$

where

$$\begin{aligned}\hat{k}_{t,T} &= \frac{v_{yy}}{v_y Y} \hat{Y}_T + \frac{v_{ya}}{v_y A} a_T + \hat{Y}_T + \theta(1+\omega) \sum_{s=t+1}^T \hat{\pi}_s \\ &= (1+w)\hat{Y}_T - \omega q_T + \theta(1+\omega) \sum_{s=t+1}^T \hat{\pi}_s\end{aligned}\tag{B.5}$$

and $a_t \equiv \ln A_t/\bar{A}$, and $\omega q_t \equiv \phi(1+\nu)a_t$. Also

$$\hat{F}_t + \frac{1}{2}\hat{F}_t^2 + \mathcal{O}(\|\xi\|^3) = (1-\alpha\beta)E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\hat{f}_{t,T} + \frac{1}{2}\hat{f}_{t,T}^2] + \mathcal{O}(\|\xi\|^3)\tag{B.6}$$

where

$$\begin{aligned}\hat{f}_{t,T} &= -\frac{\tau}{1-\tau}\hat{\tau}_T + \hat{Y}_T + \frac{u_{cc}}{u_c C}\hat{C}_T + (\theta-1) \sum_{s=t+1}^T \hat{\pi}_s \\ &= -\frac{\tau}{1-\tau}\hat{\tau}_T + \hat{Y}_T - \bar{\sigma}^{-1}\hat{C}_T + (\theta-1) \sum_{s=t+1}^T \hat{\pi}_s \\ &= -\frac{\tau}{1-\tau}\hat{\tau}_T + (1-\sigma^{-1})\hat{Y}_T + \frac{\sigma^{-1}(1-\frac{\bar{Y}}{\bar{C}})}{2}\hat{Y}_T^2 + (\theta-1) \sum_{s=t+1}^T \hat{\pi}_s\end{aligned}\tag{B.7}$$

For future reference, we further define

$$\begin{aligned}k_T &\equiv (1+\omega)\hat{Y}_T - \omega q_T \\ f_T &\equiv -\frac{\tau}{1-\tau}\hat{\tau}_T + (1-\sigma^{-1})\hat{Y}_T + \frac{\sigma^{-1}(1-\frac{\bar{Y}}{\bar{C}})}{2}\hat{Y}_T^2\end{aligned}$$

We use equations (B.4) and (B.6) to get a second order expansion of the right hand side of (B.1):

$$\begin{aligned}\hat{K}_t - \hat{F}_t &= (1-\alpha\beta)E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(\hat{k}_{t,T} - \hat{f}_{t,T} + \frac{1}{2}(\hat{k}_{t,T}^2 - \hat{f}_{t,T}^2))] - \frac{1}{2}(\hat{K}_t^2 - \hat{F}_t^2) + \mathcal{O}(\|\xi\|^3) \\ &= (1-\alpha\beta)E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(\hat{k}_{t,T} - \hat{f}_{t,T} + \frac{1}{2}(\hat{k}_{t,T}^2 - \hat{f}_{t,T}^2))] - \frac{1}{2}(\hat{K}_t - \hat{F}_t)(\hat{K}_t + \hat{F}_t) + \mathcal{O}(\|\xi\|^3) \\ &= (1-\alpha\beta)E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(\hat{k}_{t,T} - \hat{f}_{t,T} + \frac{1}{2}(\hat{k}_{t,T}^2 - \hat{f}_{t,T}^2))] - \frac{1}{2}(1-\alpha\beta)\frac{\alpha(1+\omega\theta)}{1-\alpha}\hat{\pi}_t Z_t + \mathcal{O}(\|\xi\|^3)\end{aligned}\tag{B.8}$$

where

$$Z_t = \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\hat{k}_{t,T} + \hat{f}_{t,T}]$$

The last expression was derived by substituting $(\hat{K}_t - \hat{F}_t)$ from equation (B.3), to a first order. Note we can further expand the first term of equation (B.8):

$$\begin{aligned} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(\hat{k}_{t,T} - \hat{f}_{t,T})] &= E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(\hat{k}_T - \hat{f}_T)] + (1 + \omega\theta) E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \sum_{s=t+1}^T \hat{\pi}_s \\ &= E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(\hat{k}_T - \hat{f}_T)] + \frac{(1 + \omega\theta)}{1 - \alpha\beta} \sum_{T=t+1}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T \end{aligned} \quad (\text{B.9})$$

The last part follows since

$$\begin{aligned} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \sum_{s=t+1}^T \hat{\pi}_s &= E_t \{ \alpha\beta \hat{\pi}_{t+1} + (\alpha\beta)^2 [\hat{\pi}_{t+1} + \hat{\pi}_{t+2}] + \dots \} \\ &= \frac{1}{1 - \alpha\beta} E_t \sum_{T=t+1}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T \end{aligned}$$

Also, note that

$$\begin{aligned} \frac{1}{2} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\hat{k}_{t,T}^2 - \hat{f}_{t,T}^2) &= \frac{1}{2} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\hat{k}_T^2 - \hat{f}_T^2) \\ &\quad + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\theta(1 + \omega)\hat{k}_T + (1 - \theta)\hat{f}_T] \sum_{s=t+1}^T \hat{\pi}_s \\ &\quad + \frac{1}{2} (2\theta + \theta\omega - 1)(1 + \theta\omega) E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\sum_{s=t+1}^T \hat{\pi}_s \right)^2 \\ &= \frac{1}{2} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\hat{k}_T^2 - \hat{f}_T^2) + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T N_T \\ &\quad + \frac{1}{2} \frac{(2\theta + \theta\omega - 1)(1 + \theta\omega)}{1 - \alpha\beta} E_t \sum_{T=t+1}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T (\hat{\pi}_T + 2V_T) \end{aligned} \quad (\text{B.10})$$

where

$$\begin{aligned} N_t &\equiv E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\theta(1 + \omega)\hat{k}_T + (1 - \theta)\hat{f}_T] \\ V_T &\equiv E_t \sum_{s=T+1}^{\infty} (\alpha\beta)^{s-T} \hat{\pi}_s \end{aligned}$$

Then it follows that

$$\begin{aligned} \hat{K}_t - \hat{F}_t &= (1 - \alpha\beta) E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\hat{k}_T - \hat{f}_T] + (1 + \omega\theta) E_t \sum_{T=t+1}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T - \frac{1}{2} (1 - \alpha\beta) \frac{\alpha(1 + \omega\theta)}{1 - \alpha} \hat{\pi}_t Z_t \\ &\quad + \frac{1}{2} (1 - \alpha\beta) E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\hat{k}_T^2 - \hat{f}_T^2] + (1 - \alpha\beta) E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T N_T \\ &\quad + \frac{1}{2} (2\theta + \theta\omega - 1)(1 + \theta\omega) E_t \sum_{T=t+1}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T (\hat{\pi}_T + 2V_T) + \mathcal{O}(\|\xi\|^3) \end{aligned} \quad (\text{B.11})$$

This can be written recursively as

$$\begin{aligned}
\hat{K}_t - \hat{F}_t + \frac{1}{2}(1 - \alpha\beta) \frac{\alpha(1 + \omega\theta)}{1 - \alpha} \pi_t Z_t &= (1 - \alpha\beta) [\hat{k}_T - \hat{f}_T + \frac{1}{2}(\hat{k}_T^2 - \hat{f}_T^2)] + \alpha\beta(1 + \omega\theta) E_t \hat{\pi}_{t+1} \\
&+ (1 - \alpha\beta) \alpha\beta E_t \pi_{t+1} N_{t+1} \\
&+ \frac{1}{2}(2\theta + \theta\omega - 1)(1 + \theta\omega) \alpha\beta E_t \pi_{t+1} (\pi_{t+1} + 2V_{t+1}) \\
&+ \alpha\beta [\hat{K}_{t+1} - \hat{F}_{t+1} + \frac{1}{2}(1 - \alpha\beta) \frac{\alpha(1 + \omega\theta)}{1 - \alpha} \pi_t Z_{t+1}] + \mathcal{O}(\|\xi\|^3)
\end{aligned} \tag{B.12}$$

Substitute out $\hat{K}_t - \hat{F}_t$ using equation (B.3):

$$\begin{aligned}
\hat{\pi}_t + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_t^2 + \frac{1}{2}(1 - \alpha\beta) \hat{\pi}_t Z_t &= \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha(1 + \omega\theta)} [\hat{k}_T - \hat{f}_T + \frac{1}{2}(\hat{k}_T^2 - \hat{f}_T^2)] \\
&+ (1 - \alpha)\beta E_t \hat{\pi}_{t+1} + \frac{(1 - \alpha\beta)(1 - \alpha)\beta}{1 + \omega\theta} E_t \hat{\pi}_{t+1} N_{t+1} \\
&+ \frac{1}{2}(2\theta + \theta\omega - 1)(1 - \alpha)\beta E_t \hat{\pi}_{t+1} (\hat{\pi}_{t+1} + 2V_{t+1}) \\
&+ \alpha\beta E_t [\hat{\pi}_{t+1} + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_{t+1}^2 + \frac{1}{2}(1 - \alpha\beta) \hat{\pi}_{t+1} Z_{t+1}] + \mathcal{O}(\|\xi\|^3)
\end{aligned} \tag{B.13}$$

Now note that N_t can be rewritten:

$$\begin{aligned}
N_t &= \frac{1}{2} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(1 + \theta\omega)(\hat{k}_T + \hat{f}_T) + (2\theta + \theta\omega - 1)(\hat{k}_T - \hat{f}_T)] \\
&= \frac{1}{2} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(1 + \theta\omega)(\hat{k}_{t,T} + \hat{f}_{t,T}) + (2\theta + \theta\omega - 1)(\hat{k}_{t,T} - \hat{f}_{t,T})] - \frac{(2\theta + \theta\omega - 1)(1 + \omega\theta)}{1 - \alpha\beta} V_t \\
&= \frac{1}{2}(1 + \theta\omega) Z_t + \frac{1}{2}(2\theta + \theta\omega - 1) \frac{\alpha(1 + \theta\omega)}{(1 - \alpha)(1 - \alpha\beta)} \hat{\pi}_t - \frac{(2\theta + \theta\omega - 1)(1 + \omega\theta)}{1 - \alpha\beta} V_t
\end{aligned} \tag{B.14}$$

where the second term of the last expression comes from using equation (B.3), to the first order. Substituting equation (B.14) into equation (B.13) yields

$$\begin{aligned}
\hat{\pi}_t + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_t^2 + \frac{1}{2}(1 - \alpha\beta) \hat{\pi}_t Z_t &= \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha(1 + \omega\theta)} [\hat{k}_T - \hat{f}_T + \frac{1}{2}(\hat{k}_T^2 - \hat{f}_T^2)] \\
&+ (1 - \alpha)\beta E_t \hat{\pi}_{t+1} + \frac{1}{2}(1 - \alpha)\beta(1 - \alpha\beta) E_t Z_{t+1} \hat{\pi}_{t+1} \\
&+ \frac{1}{2}(2\theta + \theta\omega - 1) \alpha\beta E_t \hat{\pi}_{t+1}^2 \\
&- (2\theta + \theta\omega - 1)(1 - \alpha)\beta E_t V_{t+1} \hat{\pi}_{t+1} \\
&+ \frac{1}{2}(2\theta + \theta\omega - 1)(1 - \alpha)\beta E_t \hat{\pi}_{t+1} (\hat{\pi}_{t+1} + 2V_{t+1}) \\
&+ \alpha\beta E_t [\hat{\pi}_{t+1} + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_{t+1}^2 + \frac{1}{2}(1 - \alpha\beta) \hat{\pi}_{t+1} Z_{t+1}] + \mathcal{O}(\|\xi\|^3)
\end{aligned} \tag{B.15}$$

After some algebra and collecting terms, this can be written as

$$\begin{aligned}\hat{\pi}_t + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_t^2 + \frac{1}{2} (1 - \alpha\beta) \hat{\pi}_t Z_t &= \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha(1 + \omega\theta)} [\hat{k}_T - \hat{f}_T + \frac{1}{2} (\hat{k}_T^2 - \hat{f}_T^2)] \\ &+ \beta E_t \hat{\pi}_{t+1} + \frac{1}{2} \beta (1 - \alpha\beta) E_t \hat{\pi}_{t+1} Z_{t+1} \\ &+ \beta E_t \left[\frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_{t+1}^2 \right] + \frac{1}{2} \theta (1 + \omega) \beta E_t \hat{\pi}_{t+1}^2 + \mathcal{O}(\|\xi\|^3)\end{aligned}\tag{B.16}$$

Define $V_t \equiv \hat{\pi}_t + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_t^2 + \frac{1}{2} (1 - \alpha\beta) \hat{\pi}_t Z_t + \frac{1}{2} \theta (1 + \omega) \hat{\pi}_t^2$. Then equation (B.16) can be written as

$$V_t = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha(1 + \omega\theta)} [\hat{k}_T - \hat{f}_T + \frac{1}{2} (\hat{k}_T^2 - \hat{f}_T^2)] + \frac{1}{2} \theta (1 + \omega) \hat{\pi}_t^2 + \beta E_t V_{t+1}\tag{B.17}$$

Substituting for k_T and f_T , we arrive at the second order approximation of equation (A.23):

$$\begin{aligned}V_t &= \kappa \left\{ \hat{Y}_t + (\omega + \sigma^{-1})^{-1} \left[\frac{\tau}{1 - \tau} \hat{\tau}_t - \omega q_t \right] + \frac{1}{2} d_{yy} \hat{Y}_t^2 - \hat{Y}_t d_{y\xi} \xi_t + \frac{1}{2} d_\pi \hat{\pi}_t^2 \right\} \\ &+ \beta E_t V_{t+1} + \text{t.i.p} + \mathcal{O}(\|\xi\|^3)\end{aligned}\tag{B.18}$$

where

$$\begin{aligned}\kappa &\equiv \frac{(1 - \alpha\beta)(1 - \alpha)(\omega + \sigma^{-1})}{\alpha(1 + \theta\omega)} \\ d_{yy} &\equiv (2 + \omega - \sigma^{-1}) + \sigma^{-1} \left(1 - \frac{\bar{Y}}{C} \right) (\omega + \sigma^{-1})^{-1} \\ d_{y\xi} \xi_t &\equiv (\omega + \sigma^{-1})^{-1} \left[(1 + \omega) \omega q_t - (1 - \sigma^{-1}) \frac{\tau}{1 - \tau} \hat{\tau}_t \right] \\ d_\pi &\equiv \frac{\theta(1 + \omega)}{\kappa}\end{aligned}$$

and t.i.p. refers to terms that are not dependent on policy (i.e. terms that only involve the exogenous variables). Note that to a first order approximation, equation (B.18) reduces to equation (1), the Phillips curve, in the text:

$$\hat{\pi}_t = \kappa [\hat{Y}_t + \psi \hat{\tau}_t] + \beta E_t \hat{\pi}_{t+1} + \hat{a}_t$$

where

$$\hat{a}_t \equiv -\kappa (\omega + \sigma^{-1})^{-1} \omega q_t$$

and

$$\psi \equiv (\omega + \sigma^{-1})^{-1} \frac{\tau}{1 - \tau} > 0$$

Note the form of the Phillips curve is not specific to the assumption of a tax on profits. If we had assumed instead a tax on labor income, the above equation would still hold (although the second order approximation would differ slightly).

B.3 Derivation of Quadratic Loss Function, (5)

Note that

$$\begin{aligned}\int_0^1 v(H_t(j))dj &= \int_0^1 \frac{\lambda}{1+\nu} \left(\frac{Y_t(j)}{A_t}\right)^{\phi(1+\nu)} dj \\ &= \frac{\lambda}{1+\nu} \frac{Y_t^{1+\omega}}{A_t^{1+\omega}} \Delta_t \\ &\equiv v(Y_t)\Delta_t\end{aligned}$$

where Δ_t is the measure of price dispersion:

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta(1+\omega)} di$$

Then the utility function (A.1) can be written as

$$U \equiv E_t \sum_{t=0}^{\infty} \beta^t [u(Y_t) - v(Y_t(j))\Delta_t] \quad (\text{B.19})$$

A second order Taylor approximation of the first term gives

$$\begin{aligned}u(Y_t; \xi_t) &= \bar{u} + \bar{u}_c(Y_t - \bar{Y}) + \frac{1}{2}\bar{u}_{cc}(Y_t - \bar{Y})^2 + \mathcal{O}(\|\xi\|^3) \\ &= \bar{u} + \bar{Y}\bar{u}_c(1 + \hat{Y}_t + \frac{1}{2}\hat{Y}_t^2) - \bar{u}_c\bar{Y} + \frac{1}{2}\bar{u}_{cc}\bar{Y}^2\hat{Y}_t^2 + \mathcal{O}(\|\xi\|^3)\end{aligned}$$

Now note that

$$u_{cc} = -\frac{\tilde{\sigma}^{-1}}{c}\bar{u}_c = -\frac{\sigma^{-1}}{Y}\bar{u}_c$$

Using these expressions, $u(Y_t)$ can be written as

$$u(Y_t) = \bar{Y}\bar{u}_c[\hat{Y}_t + \frac{1}{2}(1 - \sigma^{-1})\hat{Y}_t^2] + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3) \quad (\text{B.20})$$

where t.i.p. stands for terms that are independent of policy, specifically \bar{u} . These terms can be ignored since they are not relevant for the welfare ranking of alternative policies.

A second order expansion of the second term of equation (B.19) gives

$$\begin{aligned}v(Y_t; \xi_t)\Delta_t &= \bar{v} + \bar{v}(\Delta_t - 1) + \bar{v}_y(Y_t - \bar{Y}) + \bar{v}_y(\Delta_t - 1)(Y_t - \bar{Y}) + (\Delta_t - 1)\bar{v}_\xi\xi_t \\ &\quad + \bar{v}_\xi\xi_t + \frac{1}{2}\bar{v}_{yy}(Y_t - \bar{Y})^2 + (Y_t - \bar{Y})\bar{v}_{y\xi}\xi_t + \frac{1}{2}\xi_t'\bar{v}_{\xi\xi}\xi_t + \mathcal{O}(\|\xi\|^3) \\ &= \bar{v}(\Delta_t - 1) + \bar{v}_y\bar{Y}(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2) + \bar{v}_y\bar{Y}(\Delta_t - 1)\hat{Y}_t + \bar{v}_\xi(\Delta_t - 1)\xi_t \\ &\quad + \frac{1}{2}\bar{v}_{yy}\bar{Y}^2\hat{Y}_t^2 + \bar{Y}\bar{v}_{y\xi}\hat{Y}_t\xi_t + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3)\end{aligned}$$

Now note that

$$\begin{aligned}\bar{v}_y &= \frac{1+\omega}{Y}\bar{v}, & \bar{v}_{yy} &= \frac{\omega}{Y}\bar{v}_y \\ \bar{v}_a &= -(1+\omega)\bar{v}, & \bar{v}_{ya} &= -(1+\omega)v_y\end{aligned}$$

Using these expressions, $v(Y_t; \xi_t)\Delta_t$ can be written as

$$v(Y_t; \xi_t)\Delta_t = \bar{v}_y \bar{Y} \left[\frac{\Delta_t - 1}{1 + \omega} + \hat{Y}_t + \frac{1}{2}(1 + \omega)\hat{Y}_t^2 + (\Delta_t - 1)\hat{Y}_t - \hat{Y}_t\omega q_t - \frac{\Delta_t - 1}{1 + \omega}\omega q_t \right] + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3) \quad (\text{B.21})$$

It would be useful to express $\Delta_t - 1$ in terms of $\hat{\Delta}_t$. Towards this end, note that using the price index, the price dispersion measure can be written as

$$\Delta_t = \alpha\Delta_{t-1}\Pi_t^{\theta(1+\omega)} + (1 - \alpha) \left(\frac{1 - \alpha\Pi_t^{\theta-1}}{1 - \alpha} \right)^{\frac{\theta(1+\omega)}{\theta-1}}$$

Taking a Taylor series expansion of this equation around $\bar{\Pi} = 1$ and $\bar{\Delta} = 1$ gives

$$\begin{aligned} \hat{\Delta}_t &= \alpha\hat{\Delta}_{t-1} - \frac{1}{2} \left[\alpha\theta(1 + \omega)(\theta - 2) - \frac{\alpha^2\theta(1 + \omega)(1 + \theta\omega)}{1 - \alpha} - \alpha\theta(1 + \omega)(\theta + \theta\omega - 1) \right] \hat{\pi}_t^2 + \mathcal{O}(\|\xi\|^3) \\ &= \alpha\hat{\Delta}_{t-1} + \frac{\alpha\theta(1 + \omega)(1 + \theta\omega)}{(1 - \alpha)(\theta - 1)} \frac{\hat{\pi}_t^2}{2} + \mathcal{O}(\|\xi\|^3) \end{aligned} \quad (\text{B.22})$$

This equation has no linear inflation terms, thus $\hat{\Delta}_t = \mathcal{O}(\|\xi\|^2)$ for all $t > t_0$ if $\hat{\Delta}_{t_0-1} = \mathcal{O}(\|\xi\|^2)$. It follows that $\Delta_t - 1 = \mathcal{O}(\|\xi\|^2)$ for all $t > t_0$. Substituting this into equation (B.21) gives

$$v(Y_t; \xi_t)\Delta_t = \bar{v}_y \bar{Y} \left[\frac{\hat{\Delta}_t}{1 + \omega} + \hat{Y}_t + \frac{1}{2}(1 + \omega)\hat{Y}_t^2 - \hat{Y}_t\omega q_t \right] + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3)$$

From the household's first order condition for labor, v_y can be expressed as a function of u_c :

$$v_h = \frac{(1 - \tau)(\theta - 1)}{\theta} u_c$$

Substituting this in the above equation gives

$$v(Y_t; \xi_t)\Delta_t = (1 - \Phi)\bar{u}_c \bar{Y} \left[\frac{\hat{\Delta}_t}{1 + \omega} + \hat{Y}_t + \frac{1}{2}(1 + \omega)\hat{Y}_t^2 - \hat{Y}_t\omega q_t \right] + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3) \quad (\text{B.23})$$

Combining equations (B.20) and (B.23) gives a second order expression for the utility function

$$\begin{aligned} U &= E_t \sum \beta^t \left[\bar{Y}\bar{u}_c \left\{ \hat{Y}_t + \frac{1}{2}(1 - \sigma^{-1})\hat{Y}_t^2 \right\} \right] + \text{t.i.p.} \\ &\quad - E_t \sum \beta^t \left[(1 - \Phi)\bar{Y}\bar{u}_c \left\{ \hat{Y}_t + \frac{\hat{\Delta}_t}{1 + \omega} + \frac{1}{2}(1 + \omega)\hat{Y}_t^2 - \hat{Y}_t\omega q_t \right\} \right] + \mathcal{O}(\|\xi\|^3) \\ &= \bar{Y}\bar{u}_c E_t \sum \beta^t \left[\Phi\hat{Y}_t - \frac{1}{2} \{ (\omega + \sigma^{-1}) - \Phi(1 + \omega) \} \hat{Y}_t^2 + \hat{Y}_t(1 - \Phi)\omega q_t - \frac{1 - \Phi}{1 + \omega} \hat{\Delta}_t \right] \\ &\quad + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3) \end{aligned} \quad (\text{B.24})$$

Since $\alpha < 1$, using equation (B.22), $\hat{\Delta}_t$ can be written in terms of $\hat{\pi}_t$.

$$\hat{\Delta}_t = \alpha^{t+1}\hat{\Delta}_{-1} + \frac{\alpha}{1 - \alpha}\theta(1 + \omega)(1 + \omega\theta) \sum_{s=0}^t \alpha^{t-s} \frac{\hat{\pi}_t^2}{2} + \mathcal{O}(\|\xi\|^3)$$

Multiplying this equation by β^t and summing over t gives

$$\sum_{t=0}^{\infty} \beta^t \hat{\Delta}_t = \frac{\alpha}{1 - \alpha\beta} \hat{\Delta}_{-1} + \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} \theta(1 + \omega)(1 + \omega\theta) \sum_{t=0}^{\infty} \beta^t \frac{\hat{\pi}_t^2}{2} + \mathcal{O}(\|\xi\|^3)$$

Substituting this expression into equation (B.24) gives

$$U = \bar{Y}\bar{u}_c E_t \sum \beta^t \left[\Phi \hat{Y}_t - \frac{1}{2} \{(\omega + \sigma^{-1}) - \Phi(1 + \omega)\} \hat{Y}_t^2 + \hat{Y}_t(1 - \Phi)\omega q_t \right] - \bar{Y}\bar{u}_c \frac{(1 - \Phi)\alpha\theta(1 + \omega\theta)}{(1 - \alpha)(1 - \alpha\beta)} E_t \sum \beta^t \frac{\hat{\pi}_t^2}{2} + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3) \quad (\text{B.25})$$

Multiplying the second order approximation of the aggregate supply equation, (B.18), by $\Phi\bar{Y}\bar{u}_c$ and subtracting the resulting equation from equation (B.25) gives:

$$U = -\bar{Y}\bar{u}_c E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{q_\pi}{2} \hat{\pi}_t^2 + \frac{q_y}{2} (\hat{Y}_t - \hat{Y}_t^*)^2 \right\} \quad (\text{B.26})$$

where

$$\begin{aligned} q_\pi &= \frac{\theta}{\kappa} [(\omega + \sigma^{-1}) + \Phi(1 - \sigma^{-1})] \\ q_y &= (\omega + \sigma^{-1}) + \Phi(1 - \sigma^{-1}) + \frac{\Phi\sigma^{-1}(1 - \frac{\bar{Y}}{\bar{C}})}{\omega + \sigma^{-1}} \\ \hat{Y}_t^* &= q_y^{-1} [(1 - \Phi)\omega q_t + (\omega + \sigma^{-1})^{-1} \Phi \{ (1 + \omega)\omega q_t - (1 - \sigma^{-1}) \frac{\tau}{1 - \tau} \hat{\tau}_t \}] \end{aligned}$$

Note that \hat{Y}_t^* reduces to

$$\begin{aligned} \hat{Y}_t^* &= q_y^{-1} [(1 - \Phi)\phi(1 + \nu)a_t + (\omega + \sigma^{-1})^{-1} \Phi \{ (1 + \omega)\phi(1 + \nu)a_t - (1 - \sigma^{-1}) \frac{\tau}{1 - \tau} \hat{\tau}_t \}] \\ &= -(\kappa q_y)^{-1} (\omega + \sigma^{-1}) [(1 - \Phi) + (\omega + \sigma^{-1})^{-1} \Phi(1 + \omega)] \hat{a}_t - q_y^{-1} (\omega + \sigma^{-1})^{-1} \Phi(1 - \sigma^{-1}) \frac{\tau}{1 - \tau} \hat{\tau}_t \\ &= -\varrho_a \hat{a}_t - \varrho_\tau \hat{\tau}_t \end{aligned}$$

where

$$\varrho_\tau = q_y^{-1} \Phi(1 - \sigma^{-1})\psi, \quad \varrho_a = (\kappa q_y)^{-1} [(\omega + \sigma^{-1})(1 - \Phi) + \Phi(1 + \omega)]$$

Substituting for \hat{Y}_t^* in equation (B.3), noting that $\bar{C} = \bar{Y}$ in this case, and rearranging terms gives

$$\frac{(U - \bar{U})}{\bar{C}\bar{u}_c} = -E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{q_\pi}{2} \hat{\pi}_t^2 + \frac{q_y}{2} (\hat{Y}_t - \hat{Y}_t^*)^2 \right\}$$

which, following Gali (2002), expresses the welfare approximation as a fraction of steady state consumption. Maximizing this value is the same as minimizing the loss function given by equation (5) in the text.

B.4 Derivation of Equation (28)

Assuming \hat{a}_t and u_t^τ are mean zero iid shocks, equation (??) can be written as:

$$\hat{\pi}_t = d_1(1 - \kappa\varrho_s)\hat{a}_t + d_1\kappa\psi(L + \beta d_1)u_t^\tau - E_t d_1 \sum_{j=0}^{\infty} (\beta d_1)^j \left\{ \kappa\varrho_\tau \frac{L - \theta}{1 - \theta L} u_{t+j}^\tau \right\} \quad (\text{B.27})$$

Thus, the last term needs to be simplified in order to derive equation (28) in the text. Towards this end, write this term as:

$$E_t d_1 \kappa \varrho_\tau \sum_{j=0}^{\infty} (\beta d_1)^j \frac{1}{1 - \theta L} u_{t+j-1}^\tau - E_t d_1 \theta \kappa \varrho_\tau \sum_{j=0}^{\infty} (\beta d_1)^j \frac{1}{1 - \theta L} u_{t+j}^\tau \quad (\text{B.28})$$

Notice that this can be rewritten as

$$d_1 \kappa \varrho_\tau \underbrace{E_t \sum_{j=0}^{\infty} (\beta d_1)^j \sum_{s=0}^{\infty} \theta^s u_{t+j-1-s}^\tau}_1 - d_1 \theta \kappa \varrho_\tau \underbrace{E_t \sum_{j=0}^{\infty} (\beta d_1)^j \sum_{s=0}^{\infty} \theta^s u_{t+j-s}^\tau}_2 \quad (\text{B.29})$$

Consider term 2, which can be written as:

$$\begin{aligned} & E_t \sum_{j=0}^{\infty} (\beta d_1)^j \sum_{s=0}^{\infty} \theta^s u_{t+j-s}^\tau \\ &= \underbrace{\sum_{s=0}^{\infty} \theta^s u_{t-s}^\tau}_{j=0} + \underbrace{\beta d_1 E_t \sum_{s=0}^{\infty} \theta^s u_{t+1-s}^\tau}_{j=1} + \underbrace{(\beta d_1)^2 E_t \sum_{s=0}^{\infty} \theta^s u_{t+2-s}^\tau}_{j=2} + \dots \end{aligned} \quad (\text{B.30})$$

Now notice that u_t^τ is an iid mean zero shock, so that $E_t[u_{t+j}] = 0$ for $j > 0$. This means that

$$\begin{aligned} E_t \sum_{s=0}^{\infty} \theta^s u_{t+1-s}^\tau &= \theta u_t^\tau + \theta^2 u_{t-1}^\tau + \theta^3 u_{t-2}^\tau + \dots \\ &= \theta \sum_{s=0}^{\infty} \theta^s u_{t-s}^\tau \end{aligned} \quad (\text{B.31})$$

Using this fact, expression (B.30) can be rewritten as

$$\begin{aligned} & \sum_{s=0}^{\infty} \theta^s u_{t-s}^\tau + \beta d_1 \theta E_t \sum_{s=0}^{\infty} \theta^s u_{t-s}^\tau + (\beta d_1 \theta)^2 E_t \sum_{s=0}^{\infty} \theta^s u_{t-s}^\tau + \dots \\ &= (1 + \beta d_1 \theta + (\beta d_1 \theta)^2 + \dots) \sum_{s=0}^{\infty} \theta^s u_{t-s}^\tau \\ &= (1 + \beta d_1 \theta + (\beta d_1 \theta)^2 + \dots) \frac{1}{1 - \theta L} u_t^\tau \\ &= \frac{1}{1 - \beta d_1 \theta} \frac{1}{1 - \theta L} u_t^\tau \end{aligned} \quad (\text{B.32})$$

Similarly, term 1 can be written as:

$$\begin{aligned} & E_t \sum_{j=0}^{\infty} (\beta d_1)^j \sum_{s=0}^{\infty} \theta^s u_{t+j-1-s}^\tau \\ &= \underbrace{\sum_{s=0}^{\infty} \theta^s u_{t-1-s}^\tau}_{j=0} + \underbrace{\beta d_1 E_t \sum_{s=0}^{\infty} \theta^s u_{t-s}^\tau}_{j=1} + \underbrace{(\beta d_1)^2 E_t \sum_{s=0}^{\infty} \theta^s u_{t+1-s}^\tau}_{j=2} + \underbrace{(\beta d_1)^3 E_t \sum_{s=0}^{\infty} \theta^s u_{t+2-s}^\tau}_{j=3} + \dots \\ &= \frac{L}{1 - \theta L} u_t^\tau + \frac{\beta d_1}{1 - \beta d_1 \theta} \frac{1}{1 - \theta L} u_t^\tau \end{aligned} \quad (\text{B.33})$$

Substituting expressions (B.32) and (B.33) into expression (B.29) gives

$$d_1 \kappa \varrho_\tau \left[\frac{L}{1-\theta L} u_t^\tau + \frac{\beta d_1}{1-\beta d_1 \theta} \frac{1}{1-\theta L} u_t^\tau - \frac{1}{1-\beta d_1 \theta} \frac{\theta}{1-\theta L} u_t^\tau \right] \quad (\text{B.34})$$

Thus,

$$\begin{aligned} E_t d_1 \sum_{j=0}^{\infty} (\beta d_1)^j \left\{ \kappa \varrho_\tau \frac{L-\theta}{1-\theta L} u_{t+j}^\tau \right\} \\ = d_1 \kappa \varrho_\tau \left[\frac{L}{1-\theta L} + \frac{\beta d_1 - \theta}{(1-\beta d_1 \theta)(1-\theta L)} \right] u_t^\tau \end{aligned} \quad (\text{B.35})$$

Substituting this into equation (B.27) gives equation (28) in the text.

C Derivations for the Model with a Taylor Rule

This appendix gives derivations for the model when the central bank sets the short-term interest rate, i_t , via a simple Taylor rule:

$$\hat{i}_t = \phi_y \hat{Y}_t + \phi_\pi \hat{\pi}_t \quad (\text{C.1})$$

We assume taxes are set exogenously according to

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + \iota u_{t-1}^\tau + (1 - \iota) u_t^\tau \quad (\text{C.2})$$

Equations (1), (2), (C.1), and (C.2) characterize an equilibrium for this economy. In this case the equilibrium is determinate if and only if

$$\phi_\pi + \frac{1 - \beta}{\kappa} \phi_y > 1$$

The proof is given in Woodford (2003). In what follows we will assume this condition is satisfied.

C.1 No Foresight

In this section we assume the standard timing assumption of the fiscal literature where a change in tax policy at period t alters tax rates at period t , i.e. a tax “shock.” Using the method of undetermined coefficients, the unique analytical solution to equations (C.1) - (C.2) is

$$\hat{Y}_t = a_1 \hat{\tau}_{t-1} + a_2 u_t^s + a_3 u_t^\tau \quad (\text{C.3})$$

$$\hat{\pi}_t = b_1 \hat{\tau}_{t-1} + b_2 u_t^s + b_3 u_t^\tau \quad (\text{C.4})$$

where

$$\begin{aligned} a_1 &= \frac{\kappa \rho (1 - \rho + \sigma \phi_y) \psi}{1 + \beta \rho^2 + \kappa \sigma \phi_\pi + \sigma \phi_y - \rho (1 + \beta + \kappa \sigma + \beta \sigma \phi_y)} > 0, & b_1 &= \frac{\kappa \sigma \psi \rho (\rho - \phi_\pi)}{1 + \beta \rho^2 + \kappa \sigma \phi_\pi + \sigma \phi_y - \rho (1 + \beta + \kappa \sigma + \beta \sigma \phi_y)} < 0 \\ a_2 &= \frac{1 + \sigma \phi_y}{1 + \sigma (\phi_y + \phi_\pi \kappa)} > 0, & b_2 &= \frac{-\sigma \phi_\pi}{1 + \sigma (\phi_y + \phi_\pi \kappa)} < 0 \\ a_3 &= \frac{\kappa (1 - \rho + \sigma \phi_y) \psi}{1 + \beta \rho^2 + \kappa \sigma \phi_\pi + \sigma \phi_y - \rho (1 + \beta + \kappa \sigma + \beta \sigma \phi_y)} > 0, & b_3 &= \frac{\kappa \sigma (\rho - \phi_\pi) \psi}{1 + \beta \rho^2 + \kappa \sigma \phi_\pi + \sigma \phi_y - \rho (1 + \beta + \kappa \sigma + \beta \sigma \phi_y)} < 0 \end{aligned}$$

Without foresight, a tax shock acts like a supply shock, causing output to decrease and prices to increase with a tax increase, since production is less profitable.

C.2 One Period Foresight

In this section we assume one period fiscal foresight. Using the method of undetermined coefficients, the unique analytical solution to equations (C.1) - (C.2) is

$$\hat{\pi}_t = \tilde{a}_1 \hat{\tau}_{t-1} + \tilde{a}_2 u_t^s + \tilde{a}_3 u_t^\tau + \tilde{a}_4 u_{t-1}^\tau \quad (\text{C.5})$$

$$\hat{Y}_t = \tilde{b}_1 \hat{\tau}_{t-1} + \tilde{b}_2 u_t^s + \tilde{b}_3 u_t^\tau + \tilde{b}_4 u_{t-1}^\tau \quad (\text{C.6})$$

where

$$\begin{aligned}
\tilde{a}_1 &= \frac{\kappa\rho(1-\rho+\sigma\phi_y)\psi}{1+\beta\rho^2+\kappa\sigma\phi_\pi+\sigma\phi_y-\rho(1+\beta+\kappa\sigma+\beta\sigma\phi_y)} > 0, & \tilde{b}_1 &= \frac{\kappa\sigma\psi\rho(\rho-\phi_\pi)}{1+\beta\rho^2+\kappa\sigma\phi_\pi+\sigma\phi_y-\rho(1+\beta+\kappa\sigma+\beta\sigma\phi_y)} < 0 \\
\tilde{a}_2 &= \frac{1+\sigma\phi_y}{1+\sigma(\phi_y+\phi_\pi\kappa)} > 0, & \tilde{b}_2 &= \frac{\sigma\phi_\pi}{1+\sigma(\phi_y+\phi_\pi\kappa)} > 0 \\
\tilde{a}_3 &= \frac{\kappa\psi[\kappa\sigma(1-\phi_\pi+\sigma\phi_y)-\beta(\rho-1-\sigma\phi_y)(1+\sigma\phi_y)]}{[1+\sigma(\phi_y+\phi_\pi\kappa)][1+\beta\rho^2+\kappa\sigma\phi_\pi+\sigma\phi_y-\rho(1+\beta+\kappa\sigma+\beta\sigma\phi_y)]}, & \tilde{b}_3 &= \frac{-\kappa\sigma\psi[\phi_\pi(1+\beta(1-\rho)+\beta\sigma\phi_y)-1-\sigma\phi_y]}{[1+\sigma(\phi_y+\phi_\pi\kappa)][1+\beta\rho^2+\kappa\sigma\phi_\pi+\sigma\phi_y-\rho(1+\beta+\kappa\sigma+\beta\sigma\phi_y)]} \\
\tilde{a}_4 &= \frac{\kappa(1-\rho+\sigma\phi_y)\psi}{1+\beta\rho^2+\kappa\sigma\phi_\pi+\sigma\phi_y-\rho(1+\beta+\kappa\sigma+\beta\sigma\phi_y)} > 0, & \tilde{b}_4 &= \frac{\kappa\sigma(\rho-\phi_\pi)\psi}{1+\beta\rho^2+\kappa\sigma\phi_\pi+\sigma\phi_y-\rho(1+\beta+\kappa\sigma+\beta\sigma\phi_y)} < 0
\end{aligned}$$

The realization of a tax increase causes inflation to increase and output to decrease. If $\rho_\tau = \phi_y = 0$, then news of a tax increase will always cause inflation to increase and, thus, the interest rate to increase. If $\rho_\tau > 0$ and $\phi_y > 0$, then the effects of tax news on inflation and output are ambiguous. However, it is possible to characterize how the news will affect the interest rate. Notice in this case the effect of tax news on the interest rate is given by

$$\begin{aligned}
&\frac{\phi_\pi\kappa\psi[\kappa\sigma(1-\phi_\pi+\sigma\phi_y)-\beta(\rho_\tau-1-\sigma\phi_y)(1+\sigma\phi_y)]-\phi_y\kappa\sigma\psi[\phi_\pi(1+\beta(1-\rho_\tau)+\beta\sigma\phi_y)-1-\sigma\phi_y]}{[1+\sigma(\phi_y+\phi_\pi\kappa)][1+\beta\rho_\tau^2+\kappa\sigma\phi_\pi+\sigma\phi_y-\rho_\tau(1+\beta+\kappa\sigma+\beta\sigma\phi_y)]}u_t^\tau \\
&= \frac{\kappa\psi[\sigma(\kappa\phi_\pi+\phi_y)(1-\phi_\pi+\sigma\phi_y)+\beta\phi_\pi(1+\sigma\phi_y-\rho_\tau)]}{[1+\sigma(\phi_y+\phi_\pi\kappa)][(1+\sigma\phi_y)(1-\rho_\tau\beta)+\beta\rho_\tau(1-\rho_\tau)+\kappa\sigma(\phi_\pi-\rho_\tau)]}u_t^\tau
\end{aligned}$$

The denominator is always positive while the numerator is positive if $(1+\sigma\phi_y) > \phi_\pi$ (which is plausible since for most parameter values σ is greater than 1) or if

$$1 - \phi_\pi + \sigma\phi_y < \frac{\beta\phi_\pi(1 + \sigma\phi_y - \rho_\tau)}{\sigma(\kappa\phi_\pi + \phi_y)},$$

which will hold for most parameterizations as long as κ is not too large (i.e. roughly if $\kappa < 5$). Thus, for most parameter values the interest rate will increase when there is news of a tax increase. Notice that if $\phi_y = 0$ (and hence $\phi_\pi > 1$), then news of a tax increase will always cause inflation to increase and, thus, the interest rate to increase.

D Partial Information Solution when Inflation and Taxes are Observables

This Appendix gives derivations of the solution paths of endogenous variables when inflation and taxes are observable to the monetary authority. Many of these derivations closely follow the general procedure described in Svensson and Woodford (2004). Consider the model whose structural equations are given by equation (21) in the text. Notice that equation (21) may be written in the general form

$$\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + B\hat{i}_t + \begin{bmatrix} u_{t+1} \\ 0 \end{bmatrix} \quad (\text{D.1})$$

where X_t is a vector of predetermined variables, x_t is a vector of forward-looking variables, \hat{i}_t is the central bank's policy instrument, and u_t is a vector of i.i.d. shocks with mean zero and covariance matrix Σ_u . The matrices A and B are decomposed according to X_t and x_t ,

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

The vector of observable variables, Z_t , can be written as

$$Z_t = \begin{bmatrix} \hat{\pi}_t \\ \hat{\tau}_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_D \begin{bmatrix} X_t \\ x_t \end{bmatrix} \quad (\text{D.2})$$

where D is decomposed according to X_t and x_t , $D = [D_1 \quad D_2]$. Svensson and Woodford (2004) first show that in this set-up, certainty equivalence holds and the monetary authority's policy instruments can be written as a function of the current estimates of the predetermined variables. Thus, the monetary authority will set the interest rate according to equation (20) in the text. Notice that this equation can be written as

$$\hat{i}_t = \underbrace{\begin{bmatrix} f_1 & f_3 & f_2 \end{bmatrix}}_F X_{t|t} \quad (\text{D.3})$$

Furthermore, they show that the estimate of the forward-looking variables is given by

$$x_{t|t} = GX_{t|t}, \quad \text{where } G = (A_{22} - GA_{12})^{-1}[-A_{12} + GA_{11} + (GB_1 - B_2)F] \quad (\text{D.4})$$

For the derivations of these results, refer to Svensson and Woodford (2004). Note that in this case, G is given by

$$G = \begin{bmatrix} 1 - f_1\kappa\sigma & \kappa\psi(\beta + \kappa\sigma) - \kappa[f_3\sigma + f_2\sigma(1 + \beta + \kappa\sigma)] & \kappa(\psi - f_2\sigma) \\ -f_1\sigma & \sigma[\kappa\psi - f_3 - f_2(1 + \kappa\sigma)] & -f_2\sigma \end{bmatrix}$$

Following Svensson and Woodford (2004), we solve for the central bank's estimates of the predetermined variables and use these to solve for the equilibrium paths of the forward-looking variables. Towards this end, we first guess that the solution paths of the forward-looking variables will be of the form

$$x_t = G^1 X_t + (G - G^1)X_{t|t}, \quad G^1 = \begin{bmatrix} g_{11}^1 & g_{12}^1 & g_{13}^1 \\ g_{21}^1 & g_{22}^1 & g_{23}^1 \end{bmatrix} \quad (\text{D.5})$$

for some matrix G^1 that remains to be determined. We then verify this guess by solving for the central bank's estimates of X_t and using these estimates to find the fixed point solution to G^1 . Taking G^1 as given for the time, we now proceed to solve for the monetary authority's estimates of the predetermined variables, $X_{t|t}$. Towards this end, note that it follows from combining equations (D.1)-(D.5) that

$$X_{t+1} = HX_t + JX_{t|t} + u_{t+1} \quad (\text{D.6})$$

$$Z_t = LX_t + MX_{t|t} \quad (\text{D.7})$$

where

$$H \equiv A_{11} + A_{12}G^1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (\text{D.8})$$

$$J \equiv B_1F + A_{12}(G - G^1) = 0 \quad (\text{D.9})$$

$$L \equiv D_1 + D_2G^1 = \begin{bmatrix} 0 & 0 & 1 \\ g_{11}^1 & g_{12}^1 & g_{13}^1 \end{bmatrix} \quad (\text{D.10})$$

$$M \equiv D_2(G - G^1) = \begin{bmatrix} 0 & 0 & 0 \\ 1 - f_1\kappa\sigma - g_{11}^1 & g_{12} - g_{12}^1 & \kappa(\psi - f_2\sigma) - g_{13}^1 \end{bmatrix} \quad (\text{D.11})$$

Equations (D.6-D.7) are a transition and measurement equation for a filtering problem. However, as Svensson and Woodford (2004) note, these equations are not expressed as transition and measurement equations for a standard Kalman filter problem because of the appearance of $X_{t|t}$ on the right-hand side of the measurement equation. When forward-looking variables are included as observables, this creates a simultaneity problem because the endogenous variables depend on the current estimates of the predetermined variables and these, in turn, depend on the observable variables. Ignoring this simultaneity problem for the moment, following Svensson and Woodford (2004), we guess that the Kalman filter updating equation for $X_{t+1|t+1}$ is given by

$$X_{t+1|t+1} = X_{t+1|t} + K(Z_{t+1} - LX_{t+1|t} - MX_{t+1|t+1}) \quad (\text{D.12})$$

for some Kalman gain matrix K . We then verify this guess by solving for the matrix K . To do so, we re-define equations (D.6-D.7) to express them in a form that looks like a standard Kalman filter problem and use the results of its prediction equation to solve for the matrix K . Towards this end, we first define

$$\tilde{X}_t \equiv X_t - X_{t|t-1} \quad (\text{D.13})$$

$$\tilde{Z}_t \equiv Z_t - Z_{t|t-1} = Z_t - (L + M)X_{t|t-1} \quad (\text{D.14})$$

The last expression follows from equation (D.7) conditional on the monetary authority's information at time $t - 1$. The prediction equation can then be written as

$$X_{t|t} = X_{t|t-1} + K(L\tilde{X}_t) \quad (\text{D.15})$$

Note that

$$\tilde{Z}_t = L\tilde{X}_t + M(X_{t|t} - X_{t|t-1}) \quad (\text{D.16})$$

Substituting this expression into equation (D.15) gives

$$\tilde{Z}_t = (I + MK)L\tilde{X}_t \quad (\text{D.17})$$

Equation (D.17) is a measurement equation for a standard Kalman-filter problem for an unobservable variable \tilde{X}_t and observable variable \tilde{Z}_t . All that remains is to find the corresponding transition equation in terms of \tilde{X}_t . Towards this end, first note that conditional on the central bank's information at time t , equation (D.6) is

$$X_{t+1|t} = (H + J)X_{t|t} \quad (\text{D.18})$$

Subtracting equation (D.18) from equation (D.6) and using equation (D.15) gives the transition equation:

$$\tilde{X}_{t+1} = H(I - KL)\tilde{X}_t + u_{t+1} \quad (\text{D.19})$$

Thus, the prediction equation for $\tilde{X}_{t|t}$ can be written as

$$\tilde{X}_{t|t} = PL'(I + MK)'[(I + MK)LPL'(I + MK)']^{-1}(I + MK)L\tilde{X}_t \quad (\text{D.20})$$

where $P \equiv \text{Cov}[\tilde{X}_t - \tilde{X}_{t|t-1}]$. It follows that P can be solved for from

$$P = H(I - KL)P(I - KL)'H' + \Sigma_u \quad (\text{D.21})$$

Now we can verify that the optimal estimate of X_{t+1} is given by equation (D.12). To do this, we use equations (D.14) and (D.12) to express $X_{t|t}$ in terms of the prediction error \tilde{Z}_t

$$X_{t|t} = X_{t|t-1} + K(I + MK)^{-1}\tilde{Z}_t \quad (\text{D.22})$$

By comparing equations (D.20) and (D.22) and using $\tilde{X}_{t|t} = X_{t|t} - X_{t|t-1}$, we find that

$$K(I + MK)^{-1} = PL'(I + MK)'[(I + MK)LPL'(I + MK)']^{-1}$$

which implies that

$$K = PL'(LPL')^{-1} \quad (\text{D.23})$$

Substituting this solution for K in equation (D.21) gives an expression for P

$$P = H[P - PL'(LPL')^{-1}LP]H' + \Sigma_u \quad (\text{D.24})$$

Thus, for our model, P and K are given by

$$P = \begin{bmatrix} \sigma_a^2 & 0 & 0 \\ 0 & \sigma_\tau^2 & 0 \\ 0 & 0 & \frac{(g_{11}^1)^2 \sigma_a^2 \sigma_\tau^2}{(g_{11}^1)^2 \sigma_a^2 \sigma_\tau^2 + (g_{12}^1)^2 \sigma_\tau^2} \end{bmatrix}, \quad K = \begin{bmatrix} \frac{-g_{11}g_{12}\sigma_a^2}{(g_{11}^1)^2 \sigma_a^2 + (g_{12}^1)^2 \sigma_\tau^2} & \frac{g_{11}\sigma_a^2}{(g_{11}^1)^2 \sigma_a^2 + (g_{12}^1)^2 \sigma_\tau^2} \\ \frac{-g_{12}g_{13}\sigma_\tau^2}{(g_{11}^1)^2 \sigma_a^2 + (g_{12}^1)^2 \sigma_\tau^2} & \frac{g_{12}\sigma_\tau^2}{(g_{11}^1)^2 \sigma_a^2 + (g_{12}^1)^2 \sigma_\tau^2} \\ 1 & 0 \end{bmatrix} \quad (\text{D.25})$$

Until now we have ignored the simultaneity problem with equation (D.12), due to the fact that $X_{t+1|t+1}$ appears of the right hand side of the measurement equation. This implies that the updating equation is not operational. Solving for $X_{t+1|t+1}$ from equation (D.12) eliminates the simultaneity and gives

$$\begin{aligned} X_{t+1|t+1} &= (I + KM)^{-1}[(I - KL)X_{t+1|t} + KZ_{t+1}] \\ &= (I + KM)^{-1}[(I - KL)(H + J)X_{t|t} + KZ_{t+1}] \end{aligned} \quad (\text{D.26})$$

Equation (D.26) gives the solution path for $X_{t+1|t+1}$. All that remains to be determined is the matrix G^1 such that the solution path for x_t is given by equation (D.5). Following the derivations in Svensson and Woodford (2004), we find that G^1 is determined by the relation

$$\begin{aligned} G^1 &= A_{22}^{-1}\{-A_{21} + [G^1 + (G - G^1)KL]H\} \\ &= \begin{bmatrix} 1 & \kappa\psi(\beta + \kappa\sigma) - \kappa\sigma f_2(1 + \beta + \kappa\sigma) & \kappa\psi \\ 0 & -\sigma(f_2(1 + \kappa\sigma) - \kappa\psi) & 0 \end{bmatrix} \end{aligned} \quad (\text{D.27})$$

Given G^1, K, H, J, L, M from equations (D.27),(D.23), and (D.8)-(D.11), we use equation (D.26) to find the solution path of $X_{t|t}$:

$$X_{t|t} = \begin{bmatrix} \frac{\kappa\sigma_a^2(\psi - f_2\sigma)}{\phi} & \frac{-\sigma_a^2}{\phi} \\ \frac{\kappa^2\sigma_\tau^2(f_2\sigma - \psi)[f_2\sigma(1 + \beta + \kappa\sigma) - \psi(\kappa\sigma + \beta)]}{\phi} & \frac{\kappa\sigma_\tau^2[f_2\sigma(1 + \beta + \kappa\sigma) - \psi(\beta + \kappa\sigma)]}{\phi} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \hat{\tau}_t \end{bmatrix} \quad (\text{D.28})$$

where

$$\phi = (f_1\kappa\sigma - 1)\sigma_a^2 - \kappa^2\sigma_\tau^2[f_2\sigma(1 + \beta + \kappa\sigma) - \psi(\beta + \kappa\sigma)][f_3\sigma + f_2\sigma(1 + \beta + \kappa\sigma) - \psi(\beta + \kappa\sigma)]$$

This is simply equation (23) in the text. We then find the solution paths for inflation and output from

equation (D.5)

$$\begin{bmatrix} \hat{\pi}_t \\ \hat{Y}_t \end{bmatrix} = \begin{bmatrix} 1 & \kappa\psi(\beta + \kappa\sigma) - f_2\sigma\kappa(1 + \beta + \kappa\sigma) & \kappa\psi \\ 0 & -\sigma[f_2(1 + \kappa\sigma) - \kappa\psi] & 0 \end{bmatrix} \begin{bmatrix} u_t^a \\ u_t^\tau \\ u_{t-1}^\tau \end{bmatrix} + \begin{bmatrix} -f_1\kappa\sigma & -f_3\kappa\sigma & -f_2\kappa\sigma \\ -f_1\sigma & -f_3\sigma & -f_2\sigma \end{bmatrix} \begin{bmatrix} u_{t|t}^a \\ u_{t|t}^\tau \\ u_{t-1|t}^\tau \end{bmatrix} \quad (\text{D.29})$$

This is equivalent to equation (22) in the main text. Substituting equation (D.28) into equation (D.5) gives an expression for the solution paths of output and inflation solely in terms of the predetermined variables

$$\begin{aligned} \hat{\pi}_t &= \phi_1 u_t^a + \phi_2 u_t^\tau + \frac{\nu_\tau}{1 + \frac{\kappa^2 q_\pi}{q_y}} u_{t-1}^\tau \\ \hat{Y}_t &= \phi_3 u_t^a + \phi_4 u_t^\tau - \frac{\varrho_\tau q_y + q_\pi \kappa^2 \psi}{q_y + q_\pi \kappa^2} u_{t-1}^\tau \end{aligned}$$

where

$$\begin{aligned} \phi_1 &= \frac{A}{B}, & \phi_3 &= \frac{E}{B} \\ \phi_2 &= \frac{C}{D}, & \phi_4 &= \frac{F}{D} \end{aligned}$$

$$\begin{aligned} A &= q_y(q_\pi^2 \kappa^4 (1 - \varrho_\alpha \kappa) \sigma_\alpha^2 + q_\pi q_y \kappa^2 ((2 - 2\varrho_\alpha \kappa) \sigma_\alpha^2 + \beta \kappa^2 \sigma_\tau^2 (\varrho_\tau - \psi) \psi) + q_y^2 (\sigma_\alpha^2 - \varrho_\alpha \kappa \sigma_\alpha^2 + \beta \kappa^2 \sigma_\tau (\varrho_\tau - \psi) (\varrho_\tau (1 + \beta + \kappa\sigma) - (\beta + \kappa\sigma) \psi))) \\ B &= (q_y + q_\pi \kappa^2) (q_\pi^2 \kappa^4 (\sigma_\alpha^2 + \kappa^2 \sigma_\tau^2 \psi^2) + 2q_\pi q_y \kappa^2 (\sigma_\alpha^2 + \kappa^2 \sigma_\tau^2 \psi (\varrho_\tau (1 + \beta + \kappa\sigma) - (\beta + \kappa\sigma) \psi)) + q_y^2 (\sigma_\alpha^2 + \kappa^2 \sigma_\tau^2 (\varrho_\tau (1 + \beta + \kappa\sigma) - (\beta + \kappa\sigma) \psi)^2)) \\ C &= -(q_y \kappa (\varrho_\tau q_y (1 + \beta + \kappa\sigma) + (q_\pi \kappa^2 - q_y (\beta + \kappa\sigma)) \psi) ((1 - \varrho_\alpha \kappa) q_\pi \kappa^2 \sigma_\alpha^2 (q_\pi \kappa^2 + 2q_y) + q_\pi q_y \kappa^2 \beta \kappa^2 \sigma_\tau^2 (\varrho_\tau - \psi) \psi) + q_y^2 \sigma_\alpha^2 (1 - \varrho_\alpha \kappa + \beta \kappa^2 \sigma_\tau^2 (\varrho_\tau - \psi))) \\ D &= (q_y + q_\pi \kappa^2)^2 (q_\pi^2 \kappa^4 (\sigma_\alpha^2 + \kappa^2 \sigma_\tau^2 \psi^2) + 2q_\pi q_y \kappa^2 (\sigma_\alpha^2 + \kappa^2 \sigma_\tau^2 \psi (\varrho_\tau (1 + \beta + \kappa\sigma) - (\beta + \kappa\sigma) \psi)) + q_y^2 (\sigma_\alpha^2 + \kappa^2 \sigma_\tau^2 (\varrho_\tau (1 + \beta + \kappa\sigma) - (\beta + \kappa\sigma) \psi)^2)) \\ E &= -\varrho_\alpha q_y (q_y + q_\pi \kappa^2)^2 \Sigma_\alpha^2 - \kappa (q_y \sigma_\tau^2 (\varrho_\tau (1 + \kappa\sigma) - \kappa\sigma \psi) (\varrho_{\tau\alpha u} (1 + \beta + \kappa\sigma) - \psi (\beta + \kappa\sigma)) + q_\pi^3 \kappa^4 (\sigma_\alpha^2 + \kappa^2 \sigma_\tau^2 \psi^2) + q_\pi^2 q_y \kappa^2 (2\sigma_\alpha^2 + \kappa^2 \psi \sigma_\tau^2 \varrho_\tau (1 + \beta + \kappa\sigma))) \\ F &= (q_y (\varrho_\tau (q_y + q_\pi \kappa^2)^2 (q_\pi \beta \kappa^2 - q_y (1 + \kappa\sigma - \varrho_\alpha \kappa (1 + \beta + \kappa\sigma))) \sigma_\alpha^2 + \varrho_\tau^3 q_\pi q_y^2 \beta \kappa^4 (1 + \beta \kappa\sigma)^2 \sigma_\tau - \varrho_\tau^2 q_\pi q_y \beta \kappa^4 (1 + \beta + \kappa\sigma) (q_y + 3q_y \beta - 2q_\pi \kappa^2 + 3q_y \kappa\sigma) \sigma_\tau^2 \psi) \end{aligned}$$

Table 1: Calibrated parameter and steady-state values.

β	0.99	σ^{-1}	0.157	ω	0.473
κ	0.0236	$\bar{\tau}$	0.2	θ	10
ϕ	1/3	\bar{G}/\bar{Y}	0.176	σ_a	0.02
σ_τ	0.04	ϕ_y	0.5	ϕ_π	1.5

Table 2: **Unconditional Welfare Consequences of Various Degrees of Fiscal Foresight.**

Loss Function Values (expressed as a percentage of steady state consumption)		No Foresight	1 Period Foresight	4 Period Foresight
	Discretion	0.0575727	0.057579	0.057581
	Timeless	0.044279	0.044282	0.044284
	Taylor Rule w/ \hat{Y}_t	0.076802	0.0769	0.0769
	Taylor Rule w/ $\hat{Y}_t - \hat{Y}_{t-1}$	0.076662	0.076688	0.07676
	Taylor Rule w/ \hat{y}_t	0.0763427	0.076344	0.0764
Standard Deviations		No Foresight	1 Period Foresight	4 Period Foresight
Inflation	Discretion	0.0155	0.0155	0.0155
	Timeless	0.0132	0.0132	0.0132
	Taylor Rule w/ \hat{Y}_t	0.0199	0.0199	0.0199
	Taylor Rule w/ $\hat{Y}_t - \hat{Y}_{t-1}$	0.0199	0.0199	0.0199
	Taylor Rule w/ \hat{y}_t	0.0198	0.0198	0.0199
Output Gap	Discretion	0.158136	0.158145	0.158157
	Timeless	0.15404	0.154058	0.154076
	Taylor Rule w/ \hat{Y}_t	0.0284589	0.02846	0.02846
	Taylor Rule w/ $\hat{Y}_t - \hat{Y}_{t-1}$	0.028129	0.02813	0.02813
	Taylor Rule w/ \hat{y}_t	0.026087	0.02609	0.0261
Output	Discretion	0.1906	0.1906	0.1906
	Timeless	0.1807	0.1808	0.1808
	Taylor Rule w/ \hat{Y}_t	0.0043	0.0044	0.0044
	Taylor Rule w/ $\hat{Y}_t - \hat{Y}_{t-1}$	0.0047	0.0047	0.0048
	Taylor Rule w/ \hat{y}_t	0.0067	0.0067	0.0068

Table 3: **Conditional Welfare Consequences of Various Degrees of Fiscal Foresight.**

(+) indicates welfare conditional at time t on the history being the steady state with positive disturbances (one standard deviation from the mean) occurring at period $t - 1$. (-) indicates welfare conditional at time t on the history being the steady state with negative disturbances (one standard deviation from the mean) occurring at period $t - 1$.

Loss Function Values (expressed as a percentage of steady state consumption)						
	No Foresight		1 Period Foresight		4 Period Foresight	
	(+)	(-)	(+)	(-)	(+)	(-)
Discretion	0.05766	0.05766	0.05805	0.0572732	0.0588	0.05654
Timeless	0.0439	0.0439	0.04448	0.04386	0.04491	0.04382
Taylor Rule w/ \hat{Y}_t	0.07691	0.07691	0.07980	0.0742	0.0799	0.0739
Taylor Rule w/ $\hat{Y}_t - \hat{Y}_{t-1}$	0.076773	0.076773	0.07754	0.07606	0.07965	0.074085
Taylor Rule w/ \hat{y}_t	0.07643	0.07643	0.07718	0.07573	0.07915	0.07387

Table 4: **Welfare Consequences of Various Observables in the VAR.**

Loss values are expressed as a percentage of steady state consumption. (+) indicates welfare conditional at time t on the history being the steady state with positive disturbances (half a standard deviation from the mean) occurring at period $t - 1$. (-) indicates welfare conditional at time t on the history being the steady state with negative disturbances (half a standard deviation from the mean) occurring at period $t - 1$.

	Uncond. Loss	Cond. Loss (+)	Cond. Loss (-)	St. Dev. $\hat{\pi}_t$	St. Dev. \hat{Y}_t
Discretion	0.057579	0.05805	0.05727	0.0155	0.1906
Inflation & Technology	0.0578	0.0582	0.0574	0.0158	0.191
Inflation & Taxes	0.0576	0.0581	0.0573	0.0155	0.1908

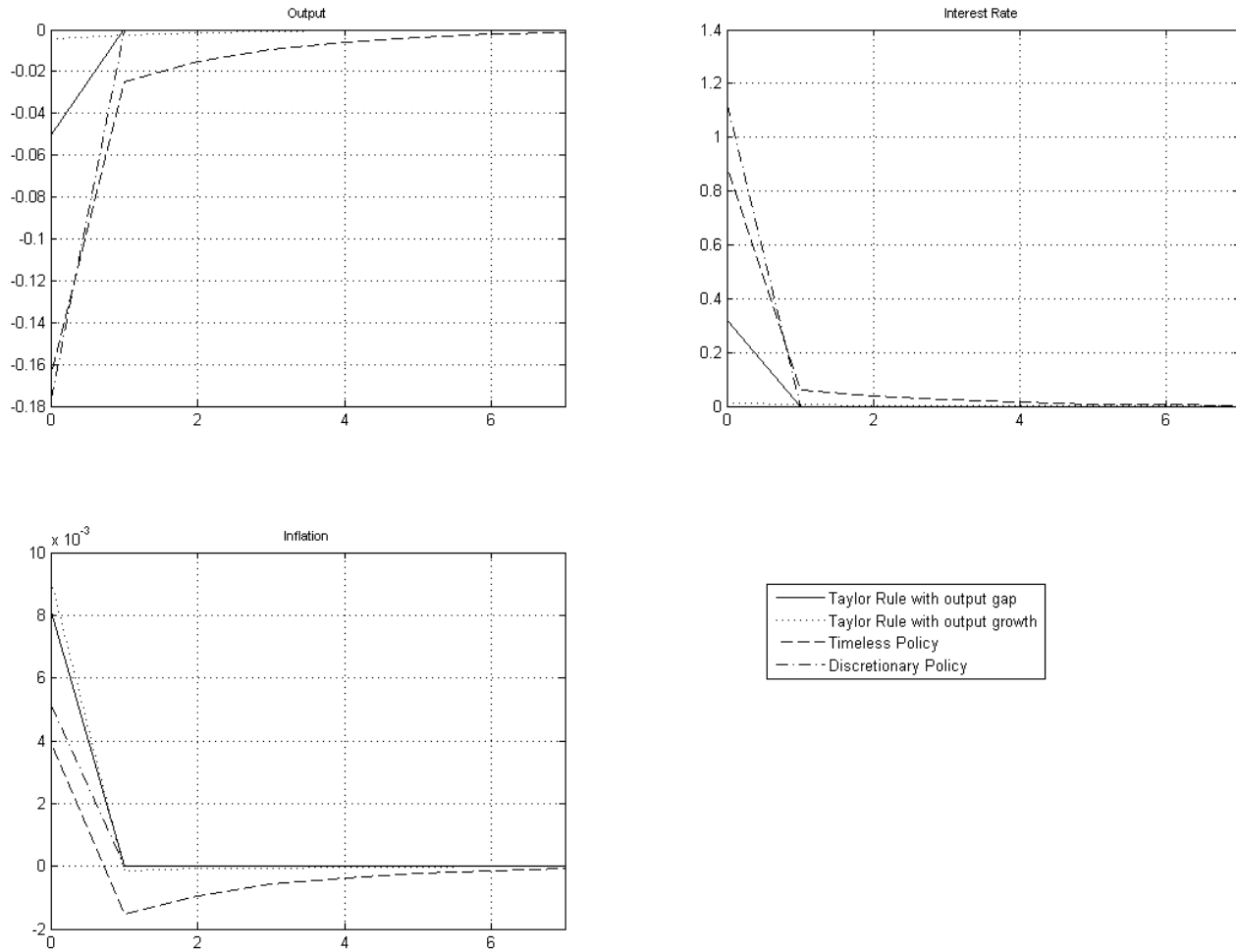


Figure 1: Impulse responses to an unanticipated 1% tax increase under various monetary policies. The response under the Taylor rule responding to output is not included as it is almost quantitatively the same as the Taylor rule responding to output growth. Solid line: Taylor rule with output gap. Dotted line: Taylor rule with output growth. Dashed line: Timeless Policy. Dotted-Dashed line: Discretionary Policy.

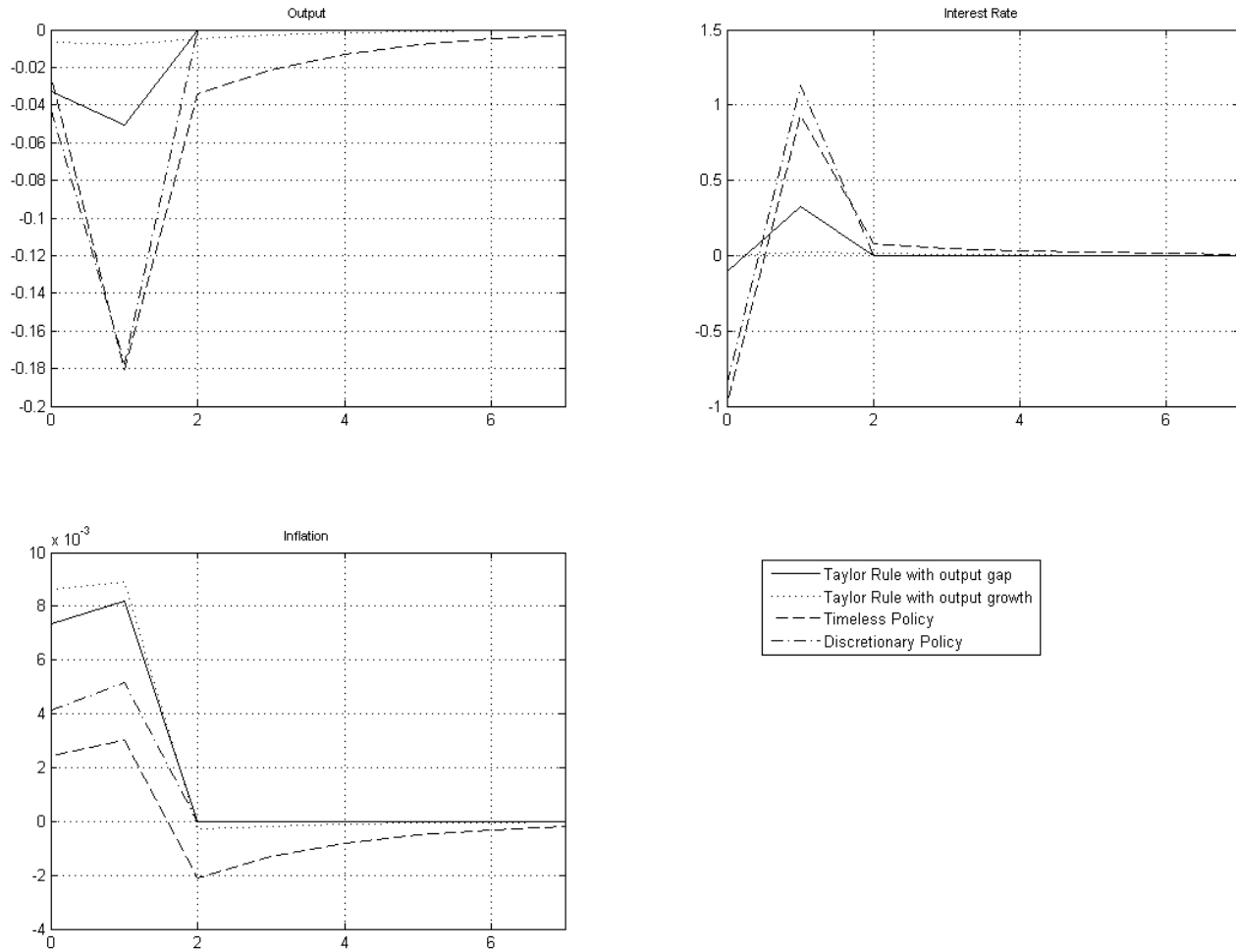


Figure 2: Impulse responses to news of a 1% tax increase one period prior to the tax change. The response under the Taylor rule responding to output is not included as it is almost quantitatively the same as the Taylor rule responding to output growth. Solid line: Taylor rule with output gap. Dotted line: Taylor rule with output growth. Dashed line: Timeless Policy. Dotted-Dashed line: Discretionary Policy.

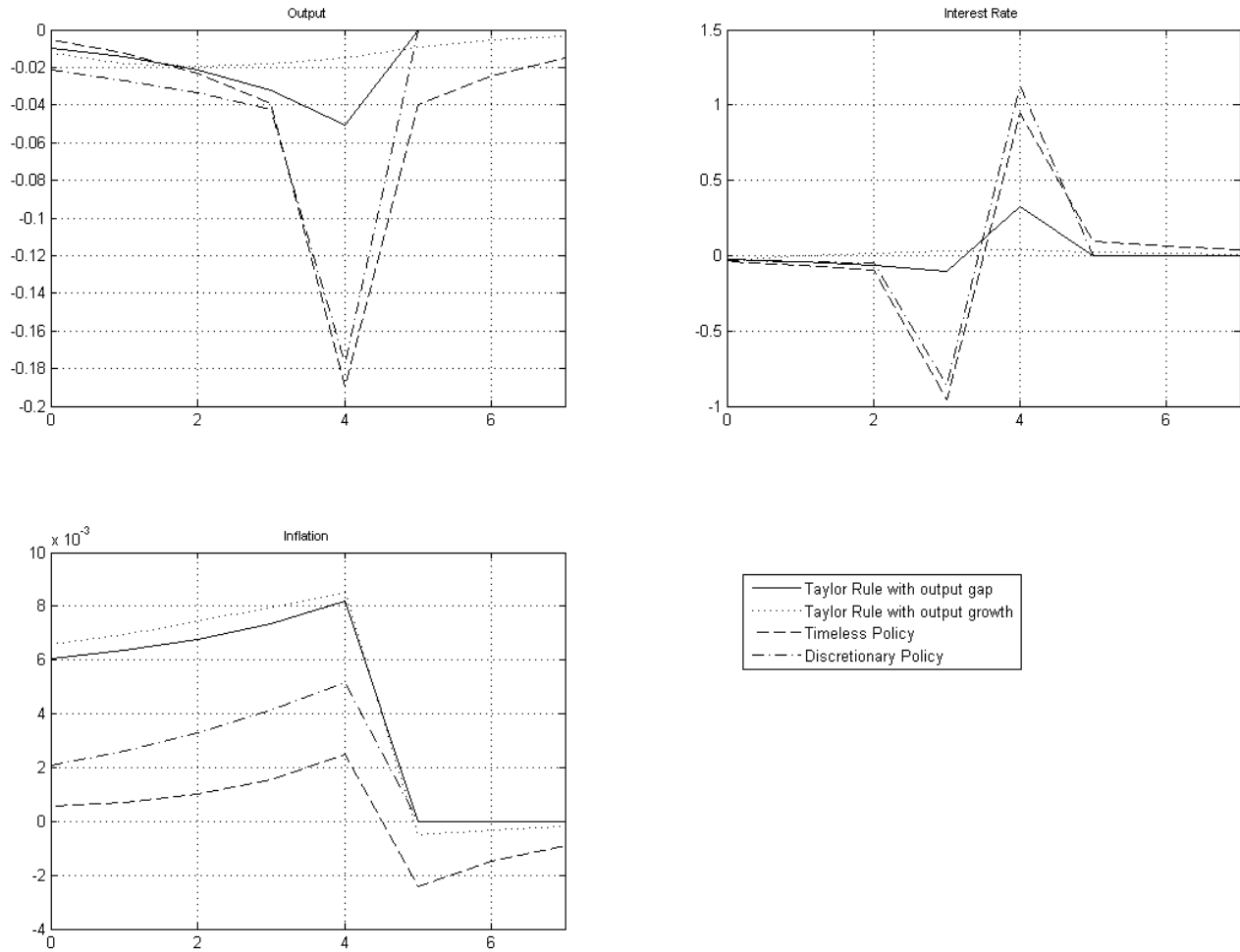


Figure 3: Impulse responses to news of a 1% tax increase four periods prior to the tax change. The response under the Taylor rule responding to output is not included as it is almost quantitatively the same as the Taylor rule responding to output growth. Solid line: Taylor rule with output gap. Dotted line: Taylor rule with output growth. Dashed line: Timeless Policy. Dotted-Dashed line: Discretionary Policy.

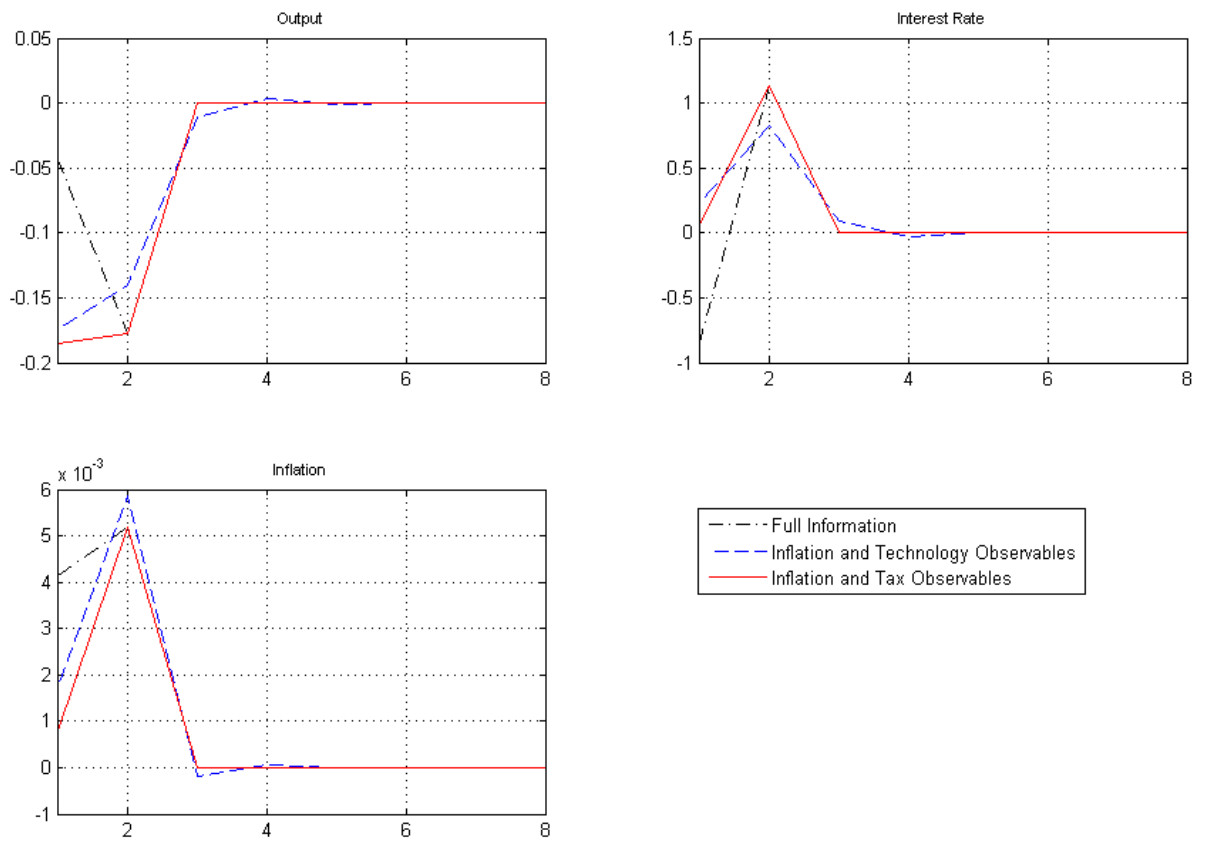


Figure 4: Impulse responses to news of a 1% tax increase one period prior to the tax change. Dotted-Dashed line: Full Information. Solid line: Tax and Inflation Observables. Dashed line: Technology and Inflation Observables.

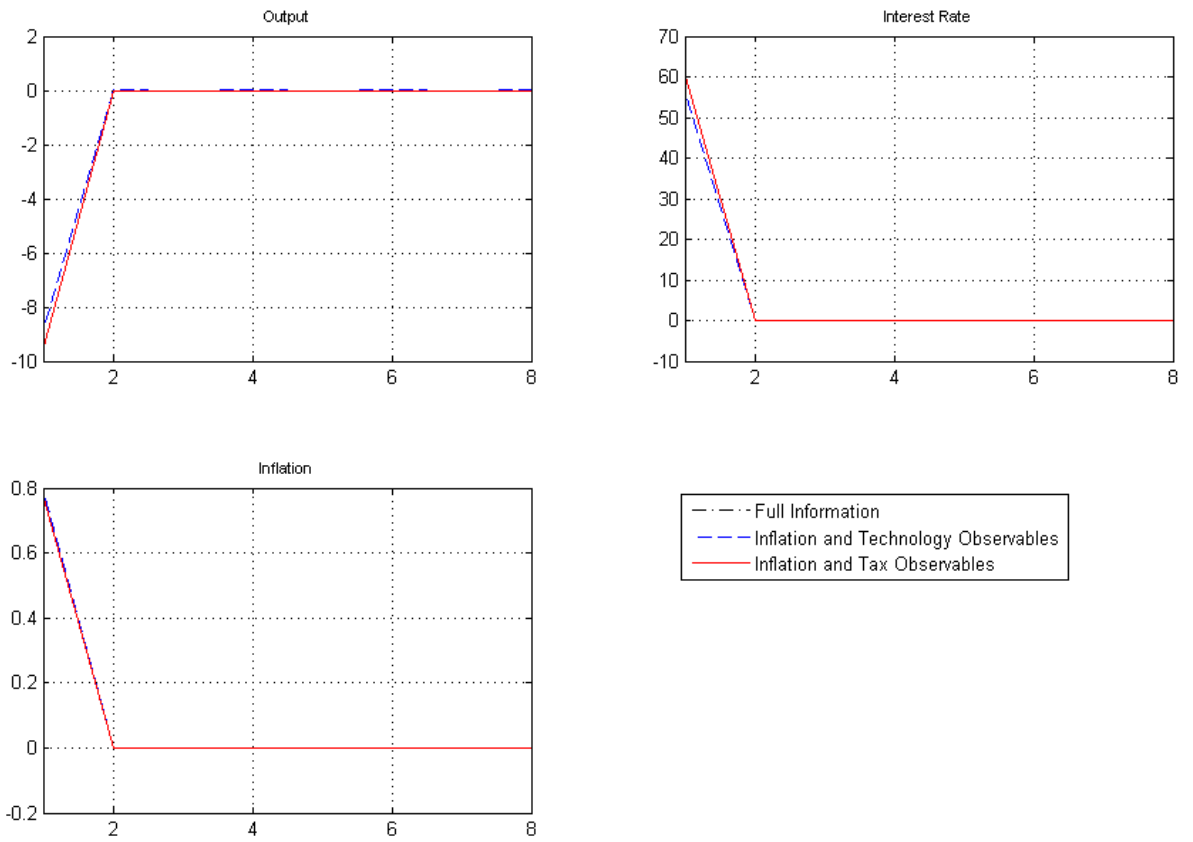


Figure 5: Impulse responses to an unanticipated 1% decrease in technology productivity. Dotted-Dashed line: Full Information. Solid line: Tax and Inflation Observables. Dashed line: Technology and Inflation Observables.

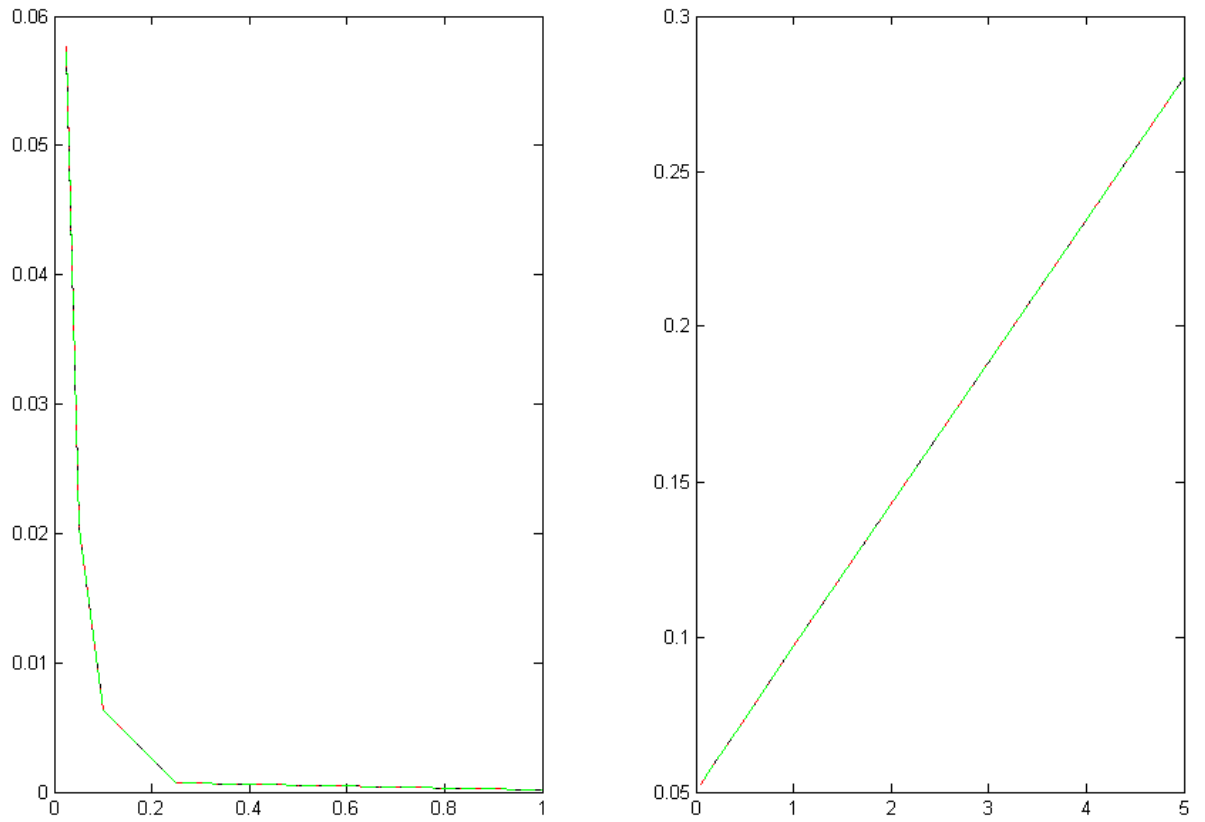


Figure 6: Unconditional welfare losses for discretionary policy. First panel: Welfare losses for as a function of κ (values on x-axis). Second panel: Welfare losses as a function of σ . Solid black line: model with no fiscal foresight and unanticipated tax changes. Red dotted dashed line: model with one period tax foresight. Green dashed line: model with four period tax foresight.