Fiscal and Trade Balances in a Model with Sticky Prices and Distortionary Taxes

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Abstract

This paper examines the interaction of fiscal and trade balances in open economices subject to monopolistic competition with sticky price-setting behavior and distortionary taxes. We find that the elasticity of net exports with respect to the real exchange rate can influence the correlation between the balances. In particular, following a shock to productivity, we find a positive correlation between trade and fiscal balances, when export elasticity is high, but a negative correlation when export elasticity is low.

Key words: sticky price setting, fiscal and trade balances
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1 Introduction

The relationship between trade and fiscal balances is of more than academic interest. For example, Bradford De Long (2004) notes that "we have a large trade deficit now–and did not back in 1997, because the federal budget deficit is much larger now than it was then." In contrast, former Undersecretary of the Treasury John Taylor (2004) argues that the trade deficit simply reflects the growth of productivity in the United States, leading to capital formation growing faster than U.S. saving. The aim of the paper is to examine the behaviour of fiscal and trade balances in an open economy subject to the distortions of monopolistic competition, sticky price setting behavior, and income taxes, with recurring productivity shocks.

In our model, the monetary authority simply targets inflation. This is consistent with recent work on monetary and fiscal interaction in open economies. Razin (2005) has argued that optimal monetary policy should put progressively more weight on inflation and less weight (or no weight) on output-gap targets as economies become more open in trade and capital flows. However, Razin eliminated the steady-state distortion of monopolistic competition by a system of taxes and subsidies, and he did not incorporate distortionary taxes and other forms of fiscal policy in his analysis. Kollmann (2004) argues for monetary rules which just respond to inflation and for a tax rate on household income that responds to public debt. He finds that this monetary/fiscal configuration yields welfare results quite close to more elaborate rules. Schmidt-Grohé and Uribe (2004) find that further emphasis on inflation by the monetary authority, beyond what is required for determinacy makes little difference for welfare, while a muted monetary response to output, with passive fiscal rules are best for welfare. Like us, Schmidt-Grohé and Uribe (2004) fully incorporate the distortionary steady-state effects of monopolistic competition in their analysis of monetary and fiscal rules.

To anticipate results, we find that the sensitivity of export demands to real exchange rate changes influences the relationship between fiscal and trade balances. In particular, in the presence of continuing productivity shocks, fiscal balances and current accounts are "twins", or positively correlated, when export demand is highly elastic; otherwise, the fiscal and current account balances are negatively correlated . In the latter case, trade deficits simply reflect the response of foreign capital to changes in domestic productivity, while fiscal balances increase with the higher tax revenue generated by rising labor income.

Our finding, that correlations of fiscal and trade balances may be positive or negative is consistent with recent work by Bussière, Fratzscher, and Müller (2005). These authors could not detect any robust empirical link between government deficits and the current account in time series studies of serveral European countries. Given that the structure of exports markets are beyond the policy scope of a small or medium size country, and that these markets are in a process of change, it should not be surprising that the link between fiscal and current account deficits change through time as well.

Erceg, Guerrieri and Gust (2004) also note that the empirical literature gives divergent estimates about the effects of fiscal deficits on the trade deficit. Like Bussière, Fratzscher, and Müller, they realized that this issue will not be settled by econometric regression results. Like us, they make use of a stochastic dynamic general equilibrium model, embedding sticky prices as well as other rigidities, to investigate the fiscal/current account linkages. They find, not surprisingly, that the trade price elasticity makes the trade balances more responsive to changes in fiscal balances, but they find that the elasticity has to be implausibly high in order for it to generate a higher response that 0.2% for a given one percent change in the fiscal deficit. Their model is more complex that the one we use here, since it contains many more distortions and rigidies that we have used. But, in contrast to their approach, we treat both fiscal and current account deficits as endogenous variables and we examine their adjustment and co-movement in response to exogenous productivity changes.

The next section describes the model as well as the monetary/fiscal policy regimes, with calibration based on Smets and Wouters (2002) open-economy version of the Euro-Area model. In section 3 we evaluate the performance of the model with impulse response function for alternative export demand regimes, one with relatively high and one with relatively low elasticity with respect to the real exchange rate. Section 4 presents accuracy tests and welfare comparisons of regimes with high and relatively low export demand. The final section concludes.

2 An Open-Economy Model with Sticky Prices

This section presents a simple model of a small open economy. It contains households which are assumed to follow the standard optimizing behavior characterized in dynamic stochastic general equilibrium models; firms with Calvo-style price-setting behavior and a monetary authority which sets the interest rate using a simple linear Taylor rule.

2.1 Households - Consumption and Labor

A representative household, at period 0, optimizes the intertemporal welfare function:

$$V = E_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t, L_t) \tag{1}$$

$$U_t(.) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varpi}}{1+\varpi} \tag{2}$$

where β is the discount factor, C_t is an index of consumption goods, L_t is labour services, σ is the coefficient of relative risk aversion and ϖ is the elasticity of marginal disutility with respect to labour supply.

The household is assumed to consume only domestically produced goods and to aggregate the bundle of differentiated goods j using a Dixit-Stiglitz aggregator:

$$C_t = \left[\int_0^1 (C_{j,t})^{\frac{d-1}{d}} dj \right]^{\frac{d}{d-1}}$$

where j denotes the domestic goods and the elasticity of substitution is given by $d > 1$. Standard cost-minimization yields demand functions:

$$C_t^j = \left(\frac{P_t^j}{P_t} \right)^{-d} C_t$$

where P_t^j is the price of each differentiated good and P_t, the aggregate price level is given by

$$P_t = \left[\int_0^1 (P_{j,t})^{1-d} dj \right]^{\frac{1}{1-d}}$$

2.2 Firms - Production and Pricing

We follow Smets and Wouters (2002) in assuming that each firm j produces differentiated goods using a Leontief technology:

$$Y_{j,t} = \min \left\{ \frac{\upsilon_t L_{j,t}}{(1-\alpha)}, \frac{K_{j,t}}{\alpha} \right\} \tag{3}$$

where υ_t is the aggregate productivity shock, which follows the following autoregressive process (in log terms):

$$\begin{aligned} \log(\upsilon_t) &= \rho \cdot \log(\upsilon_{t-1}) + \epsilon_t \\ \epsilon_t &\sim N(0, \sigma_\epsilon^2) \end{aligned} \tag{4}$$

The symbol L^j denotes the labor services hired by the firm and K^j represents the imported intermediate good which is a fixed proportion α of output. Aggregating over all firms yields aggregate supply as:

$$\begin{aligned} Y_t &= \min \left\{ \frac{\upsilon_t L_t}{1-\alpha}, \frac{K_t}{\alpha} \right\} \\ Y_t &= \left[\int_0^1 (Y_{j,t})^{\frac{d-1}{d}} dj \right]^{\frac{d}{d-1}} \\ L_t &= \int_0^1 L_{j,t} dj \\ K_t &= \int_0^1 K_{j,t} dj \end{aligned}$$

where Y is the aggregate domestic output comprising the composite bundle of differentiated goods produced by monopolistically competitive producers. The demand for good $Y_{j,t}$ is given by the following expression:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-d} Y_t$$

2.2.1 Price dispersion index and Resource Cost

Both Schmidt-Grohe and Uribe (2004) and Yun (2004) note that sticky price models with staggered pricing, creates a wedge between aggregate supply Y and aggregate demand. To see this, note first that the demand for good i is the sum of domestic and foreign demand:

$$Y_{j,t} = C_{j,t} + X_{j,t} + G_{j,t}$$

Aggregating this over the monopolistic domestic goods producers gives the following relationship between overall output, price dispersion, and the components of aggregate demand, consumption (C_t), exports (X_t) and government expenditure (G_t):

$$\begin{aligned} Y_t &= \Delta_t (C_t + X_t + G_t) \qquad (5) \\ \Delta_t &= \int \left(\frac{P_{j,t}}{P_t} \right)^{-d} dj \\ \Delta_t &\geq 1 \end{aligned}$$

where $\Delta_t \geq 1$ is a measure of relative price dispersion; with $P_{j,t}/P$ the relative price of firm j at time t.

Overall, the major implication of price stickiness is that it creates distortion. It generates real resource allocation costs leading to an overall reduction in production (and demand for labour services). Furthermore, the greater the dispersion of price in the economy, the lower the level of consumption for a given level of aggregate output and export demand. Alternatively, to maintain consumption at a particular level (for given exports and government expenditure) the greater the dispersion the greater the demand for labor and intermediate goods:

$$\begin{aligned} L_t &= \frac{(1-\alpha) Y_t}{\Delta_t \upsilon_t} \\ K_t &= \alpha \frac{Y_t}{\Delta_t} \end{aligned}$$

which in turn implies increases in disutility (reduction in welfare) and increases in the current account (and foreign debt).

2.2.2 Calvo Price Setting and Markup Distortion

We adopt a version of the Calvo (1983) staggered price system which is summarized in the equations below:

$$P^1_{j,t} = \left(\frac{P_{j,t-1}}{P_{j,t-2}}\right)^{\varkappa} P_{j,t-1}, \qquad 0 \leq \varkappa \leq 1 \tag{6}$$

$$P^2_{j,t} = \Psi \frac{Y_{j,t}MC_{j,t} + \sum_{j=1}^{\infty}\left(\frac{1}{\prod_{k=0}^{j-1}(1+R_{t+k})}\right)\xi^j Y^i_{j,t+j} MC_{j,t+j}}{Y_{j,t} + \sum_{j=1}^{\infty}\left(\frac{1}{\prod_{k=0}^{j-1}(1+R_{t+k})}\right)\xi^j Y^i_{j,t+j}} \tag{7}$$

$$\textit{where} \quad MC_{j,t} = (1-\alpha)\frac{W_t}{\upsilon_t} + \alpha P^F_t$$

$$\Psi = \frac{d}{d-1}$$

Equation (6) describes the backward pricing behavior of firms which did not receive a price-signal. For simplicity, we set the indexation parameter $\varkappa = 0$, that is, firms simply keep the price level at the previous period's level. Equation (7) is based on Calvo (1983). It describes the forward pricing behavior of the remaining firms. This framework was applied by Yun (1996) to business cycles. It represents a first-order condition from maximizing a profit function, in which a supplier will change its price at time t to maximize expected profits, based on the expected duration of the price as well as on expected demand and costs [see Woodford (2003): p. 173-203] for an extensive discussion of this framework. The term MC_t represents marginal cost which is identical across firms, P^F_t is the price of the imported intermediate goods $P^F_t = P^*_t S_t$ where P^*_t describes the price set by foreigners which is fully "passed-through" to domestic prices of imported goods. We assume an identical wage W_t, productivity factor υ_t, foreign price P^F_t, and production technology across all firms, $MC_{j,t} = MC_t$. The optimal markup factor, Ψ, equal to $\frac{d}{d-1}$, is derived from maximizing the following profit function of firm j, $\Pi_{j,t}$, with respect to the price $P_{j,t}$:

$$\Pi_{j,t} = P_{j,t}\left(\frac{P_{j,t}}{P_t}\right)^{-d} Y_t - \left(\frac{P_{j,t}}{P_t}\right)^{-d} Y_t \left[1-\alpha)\frac{W_t}{\upsilon_t} + \alpha P^F_t\right]$$

Canzoneri, Cumby and Diba (2004) note, marginal revenue divided by price, $\mathbf{d}\left[P_{j,t}Y_{j,t}\right]/P_{j,t}$, is equal to $[(d-1)/d]\,\mathbf{d}Y_{j,t}$, less than $\mathbf{d}Y_{j,t}$, with $\mathbf{d}$ representing the total differential operator for revenue $[P_{j,t}Y_{j,t}]$ and output $Y_{j,t}$. The factor $[d/(d-1)]$ is called the markup distortion created by monopolistic competition, and leads firms to produce too little.

The domestic price level for each of the differentiated goods, $P_{j,t}$ is a weighted average of a backward-looking price, $P^1_{j,t}$ with imperfect indexation, and a forward-looking component and $P^2_{j,t}$ with respective weights of ξ and $(1-\xi)$, with ξ representing the fraction of goods prices which are expected to remain unchanged; alternatively that a fraction $(1-\xi)$ of firms are forward-looking.

For simplicity, the likelihood that any price will be changed in a given period is $(1-\xi)$ and it is independent of the length of time since the price was set and the level of the current price. As Woodford (2003, p. 177) notes, while these assumptions are unrealistic, they drastically simplify equilibrium inflation dynamics as well as reduce the state-space required to solve for the dynamics. The aggregate price index is given by the following Dixit-Stiglitz aggregator:

$$P_t = \left[\xi \left(P_{t-1}\right)^{1-d} + (1-\xi)\left(P_t^2\right)^{1-d}\right]^{\frac{1}{1-d}} \tag{8}$$

Note that the lagged aggregate price P_{t-1} in equation (8) replaces $P_{j,t-1}$, which appears in equation (6).

Equation (8) may also be expressed in the following way:

$$1 = \xi\left[1+\pi_t\right]^{d-1} + (1-\xi)\left[p_t^*\right]^{1-d}$$

where p_t^* is the relative price $(P_{j,t}^2/P_t)$, and $\pi_t = ((P_t - P_{t-1})/P_{t-1})$ is the aggregate inflation between periods $t-1$ and t. Yun (2004) rewrites the dispersion index, in terms of Calvo relative prices, as the following law of motion:

$$\Delta_t = (1-\xi)\left[p_t^{j*}\right]^{-d} + \xi\left[1+\pi_t\right]^d \cdot \Delta_{t-1} \tag{9}$$

where p_t^{j*} is the relative price (P_t^{j2}/P_t), and $\pi_t = ((P_t - P_{t-1})/P_{t-1})$ is the aggregate inflation between periods $t-1$ and t.

Goodfriend and King (1997) point out that monetary policy cannot eliminate distortions caused by Ψ, since it is a steady state effect. Studies of optimal monetary policy, evaluating monetary policy rules which compare the dynamics of the model under sticky prices with the dynamics and welfare effects under flexible prices, follow the common practice of eliminating this steady-state distortion by assuming an optimal tax/subsidy scheme to offset the markup effect on pricing and production, in other word, $\Psi = 1$. However in this paper, following Schmitt-Grohé and Uríbe (2004), we do not eliminate this distortion.

The challenge facing us is to incorporate the forward pricing equation and the law of motion describing price dispersion (9) into the solution of the model The former implies that we need to devise a way to handle the sum of an infinite number of forward-looking variables and the later implies that we need to allow for the cost of price stickiness to affect production. Rather than work with infinite forward sums, following Schmidt-Grohé and Uribe (2004), we can retain the non-linear structure of the pricing system, while making use of a recursive framework with two auxiliary variables VN_t and VD_t, in the following way:

$$\begin{aligned} VN_t &= Y_t MC_t + \sum_{j=1}^{\infty}\left(\frac{1}{\prod_{k=0}^{j-1}(1+R_{t+k})}\right)\xi^j Y_{t+j} MC_{t+j} \\ &= Y_t MC_t + \frac{1}{(1+R_{t+1})}\xi VN_{t+1} \end{aligned} \tag{10}$$

$$\begin{aligned} VD_t &= Y_t + \sum_{j=1}^{\infty}\left(\frac{1}{\prod_{k=0}^{j-1}(1+R_{t+k})}\right)\xi^j Y_{t+j} \\ &= Y_t + \frac{1}{(1+R_{t+1})}\xi VD_{t+1} \end{aligned} \tag{11}$$

This simplification also allows us to write the first order Calvo-pricing equation in a form similar to the Euler equations as follows.

$$\frac{VN_t}{VD_t} = \frac{d}{d-1} \bullet \frac{\left[Y_t MC_t + \frac{1}{(1+R_{t+1})}\xi VN_{t+1}\right]}{\left[Y_t + \frac{1}{(1+R_{t+1})}\xi VD_{t+1}\right]} \tag{12}$$

2.3 Closure Conditions and Foreign Debt

The demand for exports is modelled as:

$$\log(X_t) = \log(\overline{X}) + \chi[\log(S_t/P_t) - \log(\overline{S}/\overline{P})] \tag{13}$$

where $\overline{X}, \overline{S}$, and $\overline{P}$ are the steady state values of exports, the nominal exchange rate, and the price level, and χ is the elasticity of aggregate exports (relative to steady state levels) with respect to the real exchange rate, S_t/P_t, relative to its steady state level. Exports thus depend on the current value of the real exchange rate, S_t/P_t. We could, of course, incorporate J-curve dynamics by putting in lags for the real exchange-rate effect on exports. Given the value of exports (X_t) and the imports of intermediate goods (K_t) the change in foreign debt F_t evolves as follows:

$$(P_t X_t - P_t^F K_t) = -S_t\left[F_t - F_{t-1}(1 + R_{t-1}^* + \Phi(F_{t-1}))\right] \tag{14}$$

As Schmidt-Grohé and Uribe (2003) note, without any further modification, the random walk property of this type of models implies an infinite unconditional variance for variables such as F and C. To induce stationarity in these variables, several options are available: endogenous discounting, adjustment costs for the accumulation of foreign debt, or the specification of debt-elastic risk premia. Schmidt-Grohé and Uribe find that all of the options deliver "virtually identical" results at business-cycle frequencies.

In this paper we induce stationarity by introducing an asset-elastic interest rate, that is we augment the interest on international asset R_t^* with a risk premium term Φ_t which has the following symmetric functional form:

$$\Phi_t = sign(F_t)\varphi[\exp(|F_t| - \overline{F}] \tag{15}$$

where $\overline{F}$ represents the steady-state value of the international asset. If the debt is greater (less) than the steady state, we assume that foreign lenders exact an international risk premium (discount). Note when $F_t = \overline{F}$ then $\Phi(F_t) = \varphi\left[e^{F_t-\overline{F}} - 1\right] = 0$. As Schmidt-Grohé and Uribe (2003) note, the value of the coefficient φ directly affects the volatility of the current account to GDP ratio, as well as consumption volatility.

Introducing a risk premium term which is a function of debt $\Phi(F_t) = \varphi\left[e^{F_t-\overline{F}} - 1\right]$ alters the typical Euler equations. In particular, the intertemporal budget equation becomes:

$$\frac{-S_t F_t}{(1 + R_t^* - \Phi(F_t))} + \frac{B_t}{(1 + R_t)} = -S_t F_{t-1} + B_{t-1} + W_t L_t - P_t C_t - Tax_t \quad (16)$$

where F is a one-period foreign bonds, B is one-period domestic bonds, S is the nominal exchange rate (defined as the home currency per unit of foreign), W is the wage rate, P is the overall price index, R^* is the foreign interest rate, R the domestic interest rate.

2.4 Monetary and Fiscal Policy

We assume that the central bank follows a very simple Taylor (1993) rule aimed solely at inflation stabilization:

$$\overline{R} = R^* + \phi_\pi(\pi_t - \widetilde{\pi}), \quad \phi_\pi > 1$$

and actual interest rate follows the following partial adjustment mechanism:

$$R_t = \theta R_{t-1} + (1 - \theta)\overline{R} \quad (17)$$

The target rate of inflation, is simply zero, $\widetilde{\pi} = 0$.

We also assume that government expenditures are pre-set, with $G = \overline{G}$ and taxes are levied and collected on labor income at each period t:

$$Tax_t = \tau \cdot W_t L_t \quad (18)$$

where τ is the tax rate on labor income. Government debt hence evolves according to the following equation:

$$P_t\overline{G} - Tax_t = B_t - B_{t-1}(1 + R_{t-1}) \quad (19)$$

Maximizing utility subject to the budget constraint, with respect to C_t, L_t, B_t, and F_t yields the aggregate first-order Euler equations:

$$\frac{C_t^{-\sigma}}{P_t} = \lambda_t \quad (20)$$

$$\beta \mathbf{E}_t \lambda_{t+1} = \frac{\lambda_t}{(1 + R_t)} \quad (21)$$

$$\frac{\lambda_t S_t}{(1 + R_t^* - \Phi(F_t))}\left[\frac{(1 + R_t^* + \Phi(F_t)) - F_t\Phi^{'}(F_t)}{(1 + R_t^* - \Phi(F_t))}\right] = \beta \mathbf{E}_t\left(\lambda_{t+1} S_{t+1}\right) \quad (22)$$

$$[1 - \tau]\lambda_t W_t = L_t^{\varpi} \quad (23)$$

where $\mathbf{E}_t$ is the expectations operator conditional on information available at time t.

3 Model Solution

To solve the model, we need to determine decision rules for consumption C_t, exchange rate S_t, as well as for the numerator and denominator of the forward-looking Calvo price, VN_t and VD_t such that the three intertemporal Euler equation errors, given below are minimised:

$$\epsilon_t^C = \frac{C_t^{-\sigma}}{P_t(1+R_t)} - \beta E\left(\frac{C_{t+1}^{-\sigma}}{P_{t+1}}\right)$$

$$\epsilon_t^S = \frac{C_t^{-\sigma} S_t}{P_t(1+R_t^* + \Phi(F_t))}\left[\frac{(1+R_t^*+\Phi(F_t)) - F_t\Phi'(F_t)}{(1+R_t^*+\Phi(F_t))}\right] - \beta E\left(\frac{C_{t+1}^{-\sigma}}{P_{t+1}}S_{t+1}\right)$$

$$\epsilon_t^P = \frac{VN_t}{VD_t} - \Psi\frac{\left[Y_t MC_t + \frac{1}{(1+R_{t+1})}\xi VN_{t+1}\right]}{\left[Y_t + \frac{1}{(1+R_{t+1})}\xi VD_{t+1}\right]}$$

3.1 Parameters and steady-state initial values

The calibrated values are:

$\sigma = 1.5$	$\beta = 0.99$	$\varpi = 0.25$	$\alpha = 0.15$	$\varphi = .001$
$\xi = .85$	$d = 6$	$\theta = 0.95$	$\phi_\pi = 1.2$	

The values for σ, β, ϖ, α and φ are the values suggested by Smets and Wouters (2002). The Calvo pricing parameters imply a gross mark-up rate of 1.2. The Taylor rule parameters are within the range typically used.

Using the normalisation, ($\upsilon = 1$, $\overline{S} = 1.0$), the pre-set foreign variables ($P^* = 1.0$, $\overline{R}^* = 0.04$) and the exogenous variable, ($\overline{G} = 0.15$), we solve for the initial steady state values of the other variables (C, Y, K, L, W, P, R), the implied tax rates ($\overline{\tau}$) and export value $\overline{X}$ so that initial values of foreign and domestic debt are zero ($\overline{F} = \overline{B} = 0$) and so that the Euler equations are satisfied.

The initial values of the variables appear below:

$\overline{C}$ =0.7392	$\overline{Y}$ =1.0656	$\overline{K}$ =0.1598	$\overline{L}$ =0.9058
$\overline{W}$ =0.7116	$\overline{P}$ =0.9058	$\overline{R}$ =0.01	$\overline{\tau}$ =0.2108

We can, of course, normalize on other initial conditions. In the fully stochastic simulations, in which we examine welfare based on consumption and labor, the effect of initialization is mitigated by discarding the first 25% of the sample size. We note too that this model is specified and calibrated for the case where the steady-state inflation rate is assumed to be zero.

3.2 Solution Algorithm and Decision Rules

We choose to solve the above model with a nonlinear global solution algorithm, based on the collocation projection method. We do not linearize the model, nor do we make use of first or second-order Taylor methods in the popular and widely used perturbation methods [see Collard and Julliard (2001a, 2001b) and Schmidt-Grohé and Uribe (2004a)]. There methods make use of the method of Blanchard and Khan (1985) for rational expectations models with forward and backward-looking variables. As such, they are local solutions while we use a global search method.

In this model we have five state variables, productivity index, foreign debt, the price dispersion index, domestic government debt, and the interest rate. However, some state variables are more important than others. Given the low inflation in our model, the interest rate and the price dispersion index do not change very much. We found that it makes little difference if we omit them as arguments in the decision rule.

We have the choice of specifying the decision rules for the four forward-looking variables, C, E, VN, and VD, either as a Chebyshev polynimal or as a neural netowrk. Using a Chebyshev second-order polynomial expansion, for three state variables, we have 32 parameters $(= ndcheb^{nstate}.ndecision.rule)$,where $ndcheb$, $nstate$, and $ndecision.rule$ represent the degree of the Chebyshev polynomial, the number of state variables, and the number of decision rules, respectively. For the neural network, with two neurons for each decision rule, there also 32 parameters $(= nneuron.nstate.ndecisionrule + nneuron.ndecisionrule)$, where $nnuron$ represents the number of neurons for each decision rule. In this case the number of parameters is the same, given the neural network with two neurons and a second-order polynomial expansion with three state variables. However, as the number of state variables increases, the advantage of the neural network specification over the Chebyshev orthogonal polynomial becomes more apparent. In this paper, we use the neural network specification for the functional form of the decision rules. The advanage, as noted by Sirakaya, Turnovsky, and Alemdar (2005), is that such networks, with logsigmoid functions, easily deliver control bounds on endogenous variables.

The network specification implies the following functional forms for the decision rules for C, S, VN, and VD :

$$
\begin{aligned}
\widehat{N}_{1,t}^{c} &= \psi_{11}^{c}(F_{t-1}^{*}) + \psi_{12}^{c}(\upsilon_{t}^{*}) + \psi_{13}^{c}(B_{t-1}^{*}) \\
\widehat{N}_{2,t}^{c} &= \psi_{21}^{c}(F_{t-1}^{*}) + \psi_{22}^{c}(\upsilon_{t}^{*}) + \psi_{23}^{c}(B_{t-1}^{*}) \\
\widehat{C}_{t} &= \psi_{1}^{cn}. \left(\frac{1}{1+\exp(-\widehat{N}_{1,t}^{c})} \right) + \psi_{2}^{cn}. \left(\frac{1}{1+\exp(-\widehat{N}_{2,t}^{c})} \right) \\
\widehat{N}_{1,t}^{s} &= \psi_{11}^{s}(F_{t-1}^{*}) + \psi_{12}^{s}(\upsilon_{t}^{*}) + \psi_{13}^{s}(B_{t-1}^{*}) \\
\widehat{N}_{2,t}^{s} &= \psi_{21}^{s}(F_{t-1}^{*}) + \psi_{22}^{s}(\upsilon_{t}^{*}) + \psi_{23}^{s}(B_{t-1}^{*}) \\
\widehat{S}_{t} &= \psi_{1}^{ns}. \left(\frac{1}{1+\exp(-\widehat{N}_{1,t}^{s})} \right) + \psi_{2}^{ns}. \left(\frac{1}{1+\exp(-\widehat{N}_{2,t}^{s})} \right) \\
\widehat{N}_{1,t}^{vn} &= \psi_{11}^{vn}(F_{t-1}^{*}) + \psi_{12}^{vn}(\upsilon_{t}^{*}) + \psi_{13}^{vn}(B_{t-1}^{*}) \\
\widehat{N}_{2,t}^{vn} &= \psi_{21}^{vn}(F_{t-1}^{*}) + \psi_{22}^{vn}(\upsilon_{t}^{*}) + \psi_{23}^{vn}(B_{t-1}^{*}) \\
\widehat{VN}_{t} &= \psi_{1}^{n,vn}. \left(\frac{1}{1+\exp(-\widehat{N}_{1,t}^{vn})} \right) + \psi_{2}^{n,vn}. \left(\frac{1}{1+\exp(-\widehat{N}_{2,t}^{vn})} \right) \\
\widehat{N}_{1,t}^{vd} &= \psi_{11}^{vd}(F_{t-1}^{*}) + \psi_{12}^{vd}(\upsilon_{t}^{*}) + \psi_{13}^{vd}(B_{t-1}^{*}) \\
\widehat{N}_{2,t}^{vd} &= \psi_{21}^{vd}(F_{t-1}^{*}) + \psi_{22}^{vd}(\upsilon_{t}^{*}) + \psi_{23}^{vd}(B_{t01}^{*}) \\
\widehat{VD}_{t} &= \psi_{1}^{n,vd}. \left(\frac{1}{1+\exp(\widehat{-N}_{1,t}^{vd})} \right) + \psi_{2}^{n,vd}. \left(\frac{1}{1+\exp(-\widehat{N}_{2,t}^{vd})} \right)
\end{aligned}
$$

The projection method we use involves a search over a wide grid for the state variables, in order to find the values of the coefficients in the decision rules. The search involes a minimization of the Euler equation errors based on a weighted value of the residuals. Given the nonlinear specification, it is difficult to interpret the magnitudes or signs of the coefficients in the neural network system. So we will not present the estimates of the coefficients given by the projection method. We will instead focus on the economic information available from the impulse response and the stochastic simulations.

4 Impulse Response Analysis

To make sure that the calibrated model is stable, and makes sense economically, it is useful to do impulse response analysis. In this case, we set the shock to the log of the productivity coefficient, υ_t, at .05, for period 1, and zero thereafter:

$$
\begin{aligned}
\log(\upsilon_t) &= \rho \cdot \log(\upsilon_{t-1}) + \epsilon_t \\
\epsilon_t &= 0.1, \quad t = 1 \\
\epsilon_t &= 0, \quad t > 1
\end{aligned}
$$

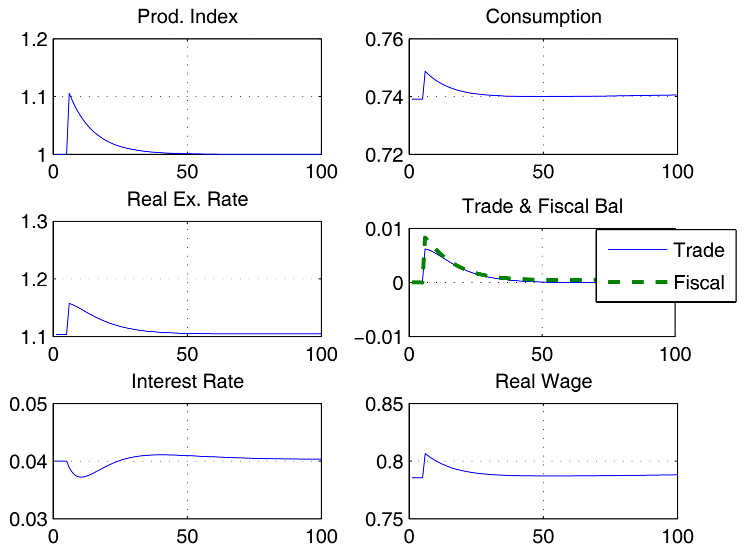

Figure 1: Impulse Response Paths with High Export Elasticity

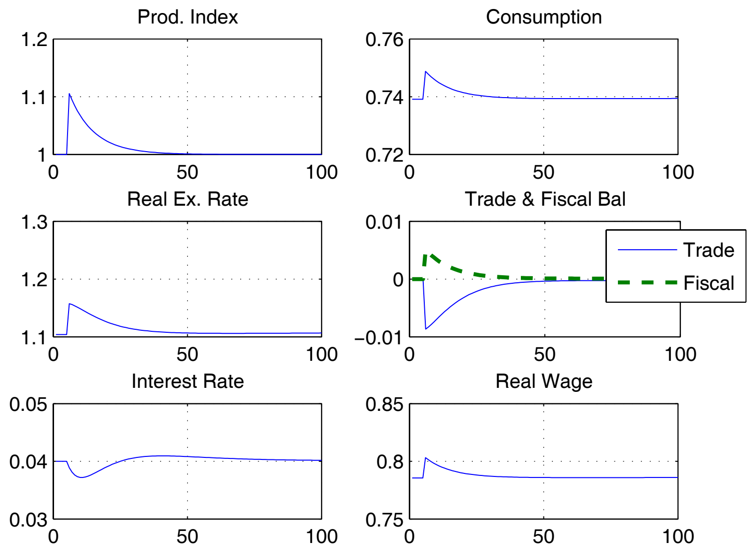

Figure 2: Impulse Response Paths with Low Export Price Elasticity

4.1 Response with High Export Elasticity

Figure (1) pictures the paths of consumption, the real exchange rate, the trade balance (exports less imports) and fiscal balance (revenue less expenditure), the interest rate and the real wages, for a 10 percent productivity shock, under the assumption of relatively high elasticity of exports with respect to the real exchange rate ($\chi = 2.0$). We see familiar results. A temporary increase in the productivity index leads to a temporary increase in consumption, the real exchange rate and real wages, a fall in the interest rate and a rise in the fiscal balance. We see that the trade balance also rises. With a relatively strong real exchange rate elasticity, exports rise more than the imports (due to the rising output), so that the current and fiscal accounts become positively correlated.

4.2 Response with Low Export Elasticity

Figure (2) pictures the same variables under the assumption of a relatively low export elasticity ($\chi = 0.2$). We see one major difference between (2) and (1). The trade balance now falls after the productivity shock. The rise in imported intermediate goods, K, is no longer offset by an increase in export demand, so that the productivity increase generates opposite reactions in the fiscal and current-account balances.

The quantitative differences between the two cases are shown in Figure (3). We see that real wages and the fiscal balance are slightly higher when exports have a very high real exchange-rate elasticity.

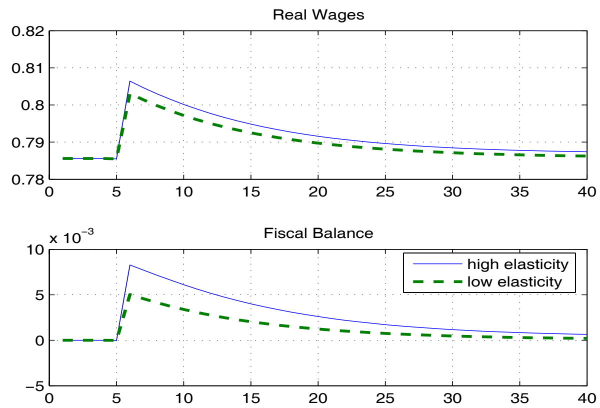

Figure 3: Real Wage and Fiscal Balances Under Alternative Assumptions

5 Stochastic Simulations

This section takes up the accuracy measures of the model, the correlations among key macroeconomic variables, and the welfare consequences of having exports with a relatively high or relatively low exchange rate elasticity.

5.1 Accuracy Assessment

Before proceeding to our analysis of the correlations of key macroeconomic indicators, we first take up the accuracy of our simulations. We make use of the Judd-Gaspar mean absolute error measures, as well as the Den-Haan and Marcet distributions.

Judd and Gaspar (1997) suggest a transformation of the Euler equation errors in the following way:

$$
\begin{aligned}
\mathcal{L}(C_t) &= \frac{\left|\epsilon_t^C\right|}{C_t} \\
\mathcal{L}(S_t) &= \frac{\left|\epsilon_t^S\right|}{C_t} \\
\mathcal{L}(P_t) &= \frac{\left|\epsilon_t^P\right|}{C_t}
\end{aligned}
$$

If the mean absolute values of the errors, deflated by consumption, is 10^{-2}, Judd and Gaspar note that the Euler equation is accurate to within a penny per dollar of expenditure. Figure (4) pictures the distribution of the Judd-Gaspar error measures for 1000 simulations of sample length 200, under the assumption of a relatively high export price elasticity, with $\chi = 2.0$ We see that the mean error measures are less than one cent per dollar of consumption expenditure. Figure (5) pictures the corresponding error distributions for the case of $\chi = 0.2$. We see that the distributions are not markedly different.

A drawback of the Judd and Gaspar criterion is that it is not based on any statistical distribution. For this reason, we use another commonly used criterion, due to den Haan and Marcet (1994). This test is denoted $DM(m)$ and is defined as:

$$
\begin{aligned}
DM(m) &= TB'A^{-1}B \sim \chi^2(m) \\
B &= \frac{1}{T}(y'X) \\
A &= \frac{1}{T}\sum X_t X_t' \varepsilon_t^2
\end{aligned}
$$

where the vector y represent the stacked Euler equation errors (for consumption, exchange rate and price), X is the instrument matrix with m columns. Under the null hypothesis of an accurate solution, the $E(y'X) = 0$. The authors recommend the following procedure for implementing this test: calculate the DM statistics repeatedly, and compute the percentage of the DM statistics which are below the lower or above the upper five percent critical values of the $\chi^2(m)$ distribution. If these fractions are noticeably different from the expected five percent, then we have evidence for an inaccurate solution. Figure (6) and (7) shows the calculated and theoretical cumulative density function of the Den-Haan Marcet Statistics for the case of high and low elasticity of export demand. The upper and lower rejection regions are 0.01/0.08 for both cases.

5.2 Correlations

How do the correlations between key macroeconomic variables change with the value of the export price elasticity? Figure (8) pictures the corelations between fiscal and trade balances, between the real exchange rate and the trade balance, between the interest rate and the real exchange rate, and between the interest rate and fiscal balances under the assumption of a relatively high export price elasticity. We see in the lower two quadrants that the correlations are negative: high interest rates are correlated with real exchange appreciations while fiscal surpluses are correlated with falling interest rates. The upper two quadrants show relatively high positive correlations. Given the high export price elasticity, a real exchange rate depreciation leads to a higher trade balance. The fiscal and trade balances are now positively correlated. Given that positive fiscal balances lower interest rates, which in turn lead to a real depreciation, a fiscal surplus goes hand in hand with a trade or current-account surplus.

Figure (9), which gives the corresponding correlations under a relatively low export price elasticity, tells another story. While the correlations in the lower two quadrants remain negative, as above, the correlations between the real exchange rate and trade balance and the correlations between the fiscal and trade balances are now negative. The key reason is that a real exchange increase, or depreciation, leads to a deterioration in the trade balance. The depreciation in the real exchange rate increases the cost of the imports, since they are used as intermediate goods to produce domestic goods, while the export demand changes little. Thus, a fiscal surplus, which lowers interest rates and leads to a depreciation, actually worsens the current account.

5.3 Welfare Comparisons

Figure (10) pictures the welfare distributions under the assumptions of relatively high or relatively low export price elasticity. We see that the variability of the welfare distribution is higher when exports are more price elastic than less price elastic. There is opportunity for welfare gain as well as welfare loss if the exports become more price elastic, due to structural change in foreign or domestic markets.

One well-known way to evaluate alternative price elastic or price-inelastic regimes is to compare the welfare of the sticky-price and tax distorted economy

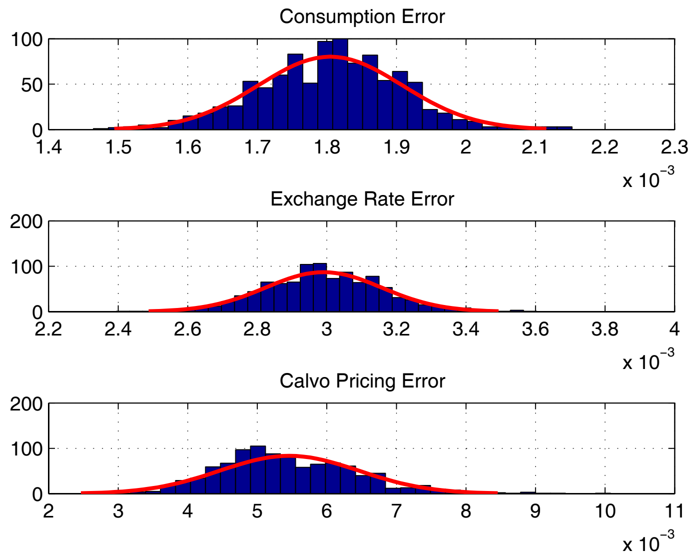

Figure 4: Judd-Gaspar Errors Statistics for High Export Price Elasticity

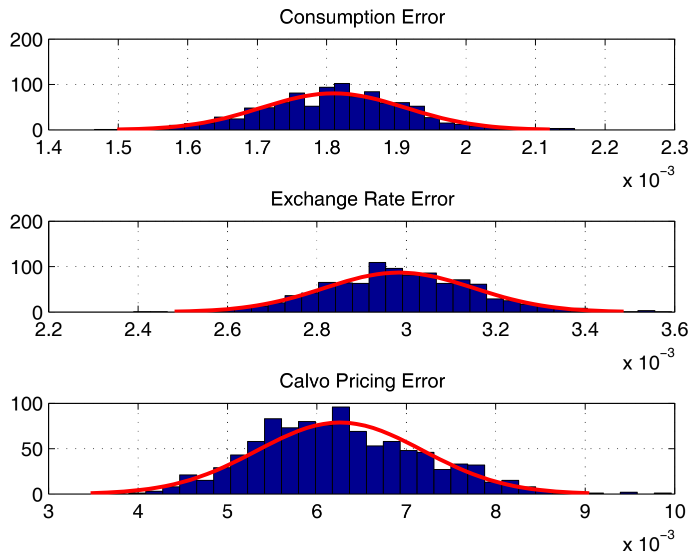

Figure 5: Judd-Gaspar Error Statistics for Low Export Price Elasticity

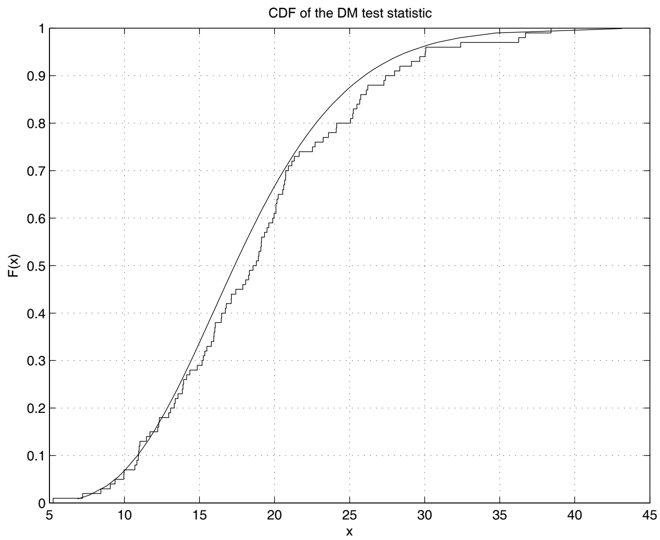

Figure 6: Case of High Export Elasticity

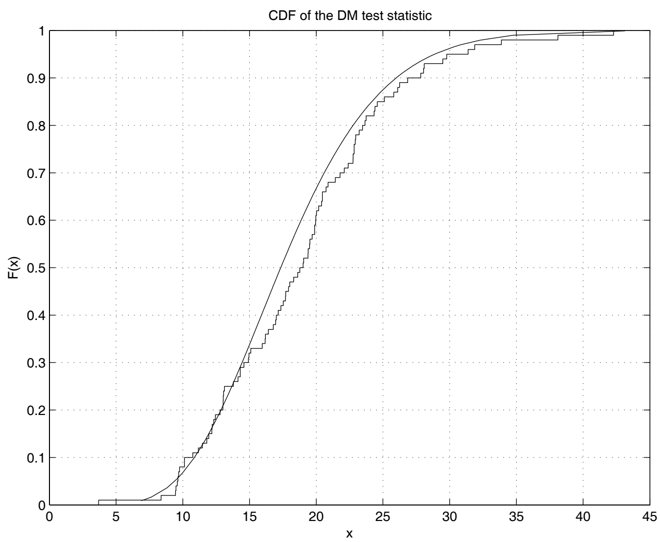

Figure 7: Case of Low Export Elasticity

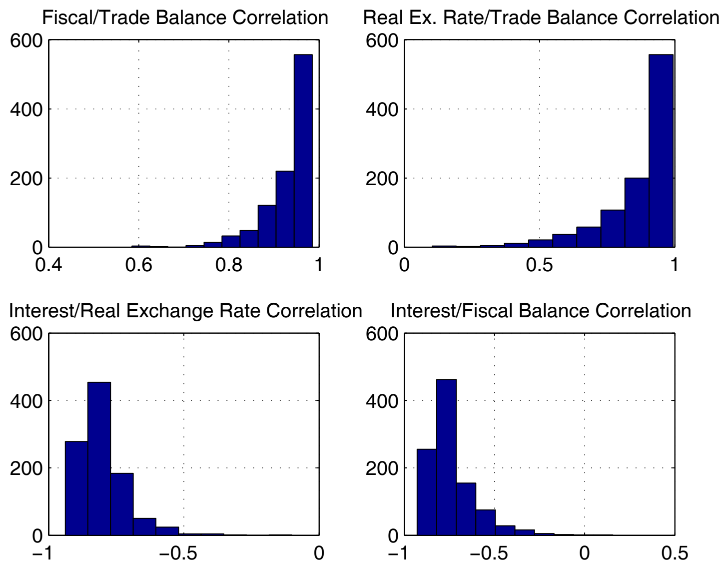

Figure 8: Macroeconomic Correlations Under High Export Price Elasticity

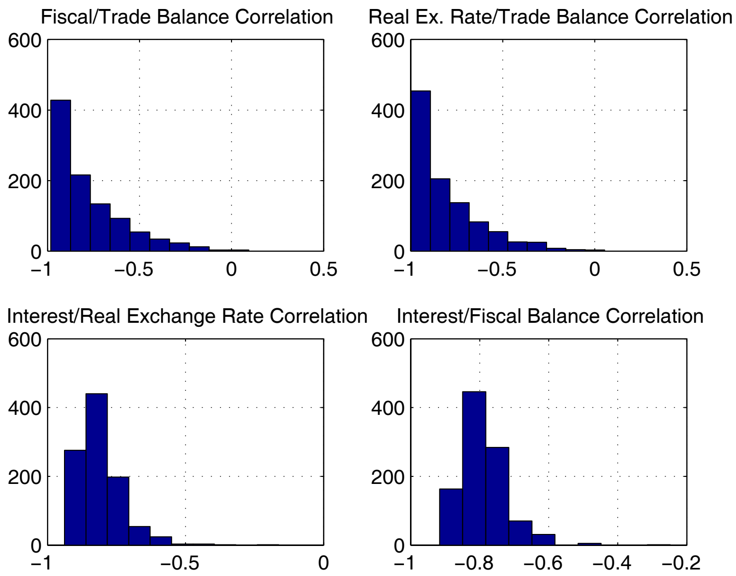

Figure 9: Macroeconomic Correlations Under Low Export Price Elasticity

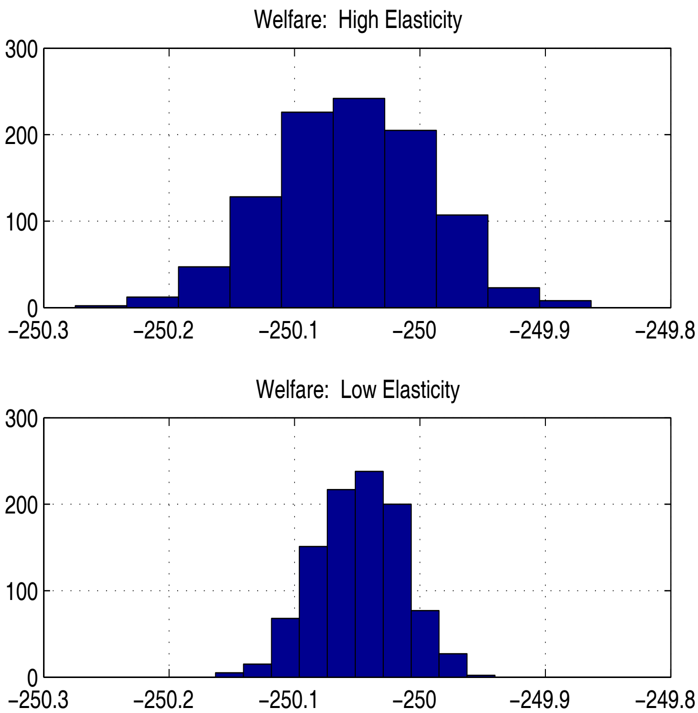

Figure 10: Welfare Distributions Under Alternative Export Price Elasticities

to the welfare of a reference regime r. The loss function of regime i can be written in the following way:

$$\ell_t^i = \frac{V_0^i - V_0^r}{V_0^r} \tag{24}$$

where V_0^r represents welfare in the reference regime r, and V_0^i the welfare in policy regime i. This loss function, of course, is measured in terms of a utility function. Following Schmitt-Grohé, Stephanie and Uribe (2004) , the differences in the two welfare indices may be re-expressed as the percentage of consumption that the household in regime i should be compensated, in order to make the household indifferent between the policy regimes i and r. With our utility function, we calculate this consumption compensation percentage in the following way:

$$
\begin{aligned}
\ell_0^{C,i} &= 100\left[1-\left(\frac{V_0^i - V_0^r}{\widetilde{C}^r}+1\right)^{\frac{1}{1-\sigma_C}}\right] \qquad (25) \\
\widetilde{C}^r &= \frac{1}{1-\sigma}\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left(C_t^r\right)^{1-\sigma}
\end{aligned}
$$

Figure (11) pictures the implied consumption compensation between the welfare distributions given in Figure (10). The value are computed with equation (25). We see that the differences amount to at most 0.3% of a unit of consumption in the reference regime of low export price elasticity. Thus the potential gains or losses are not very large if the structure of the export market changes from a relatively low to a relatively high price elasticity.

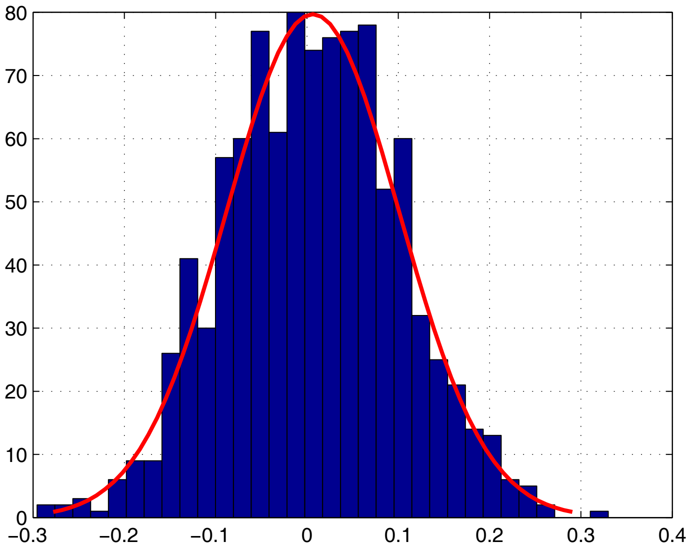

Figure 11: Percentage Consumption Compensation for Welfare Equalization

6 Conclusion

The simulations in this chapter brought up a number of interesting issues that are worthy of further exploration. First we see that fiscal and current account deficits may or may not be twins. If there is strong productivity growth and low export-price sensitivity, the current account deficit will likely increase, while the fiscal deficit will shrink. With a high export price elasticity, however, the current account deficit will shrink in tandem with the fiscal deficit as the exchange rate depreciates.

The model incorporates many of the distortions and stickiness popular in the "new neoclassical synthesis" or "new open economy macroeconomics", such as monopolistic competition, sticky price setting behavior, and distortionary taxes. However, we could have added further sources of stickiness, such as imperfect exchange-rate pass through, and sticky wage setting behavior, as well as habit persistence in consumption. We could even allow a given percentage of consumers to be non-Ricardian rule-of-thumb consumers. All of these assumptions would lower welfare but allow more scope for monetary or fiscal stabilization policy.

When all is said and done, it is better to have exports which have a high or a low price elasticity in foreign markets? In this simple setting, we assume that the export growth or volatility does not feed back into any productivity change for the home country. We assume the same structure of underlying productivity shocks driving the model, whether exports are fixed or variable. This is a drawback, of course, since exporting does generate learning effects which improve domestic productivity.

We conclude with a recurring theme. As an economy becomes more open, there are opportunities for foreign borrowing or lending, by which consumers may offset the losses of domestic distortions. We see in this paper another benefit of increasing openness or globalization. By exporting to markets where demand is highly price elastic, an economy may be able to import a degree of price flexibility through trade. This price flexibility can, of course, feed back into greater flexibility in domestic markets and thereby further improve welfare. In short, the monopolistic markup factors and the degree of price stickiness may become endogenous. Exporting to a market with greater price flexibility may be a backdoor way to import greater price flexibility and lower monopolistic distortions in the domestic market.

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