

# Measuring the Natural Rate of Interest

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## Abstract

The natural rate of interest – the real interest rate consistent with output equaling its natural rate and stable inflation – plays a central role in macroeconomic theory and monetary policy. Estimation of the natural rate of interest, however, has received little attention. We apply the Kalman filter to estimate jointly time-varying natural rates of interest and output and trend growth. We find a close link between the natural rate of interest and the trend growth rate, as predicted by theory. Estimates of the natural rate of interest, however, are very imprecise and subject to considerable real-time measurement error.

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# 1 Introduction

Since Wicksell (1936), the natural rate of interest – the real short-term interest rate consistent with output equaling its natural rate and constant inflation – has played a central role in macroeconomic and monetary theory. The natural or “equilibrium” real interest rate provides a benchmark for measuring the stance of monetary policy, with policy expansionary (contractionary) if the short-term real interest rate lies below (above) the natural rate. This role is illustrated clearly in monetary policy rules such as the Taylor (1993) rule, according to which the real interest rate exceeds the natural rate when inflation exceeds its target rate and vice versa, all else equal. Given the central position of the natural rate in the policy rule literature, the problem of real-time estimation of the natural rate of interest has received surprisingly little attention (exceptions include Rudebusch (2001) and Orphanides and Williams (2002)).<sup>1</sup> In this note, we address this issue by jointly estimating the natural rates of interest and output, and trend growth over the past 40 years of U.S. data using the Kalman filter. A key finding is that estimates of the time-varying natural rate of interest, like those of the the natural rate of unemployment and output (Staiger, Stock and Watson (1997), Orphanides and van Norden (2002)) are quite imprecise.

## 2 An Empirical Framework for Estimating the Natural Rate

Our basic approach to estimating the natural rate of interest is closely related to that of Staiger et al. (1997), Gordon (1998), and Laubach (2001), who apply the Kalman filter to estimate the the natural rate of unemployment or NAIRU.<sup>2</sup> We compare our Kalman filter estimates to those based on univariate methods designed to extract the lower frequency component from a time series.

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<sup>1</sup>See also Bomfim (1997), who estimates the natural rate of interest using the Federal Reserve Board’s now decommissioned MPS model and Kozicki and Tinsley (2001) and Bomfim (2001), who use forward rates derived from Treasury bonds and proxies of long-run inflation expectations to estimate investors’ estimates of the long-run natural rate.

<sup>2</sup>Rotemberg and Woodford (1997) and Neiss and Nelson (2001) follow a complementary approach and compute the higher frequency component of the natural rate, what Woodford (2000) terms the Wicksellian natural rate, from tightly structured macroeconomic models using detrended data. This latter approach yields the period-by-period movements in the real interest rate needed to attain constant inflation, abstracting from any lower frequency drift in the natural rate of interest.

As is clear from its definition, the natural rate of interest is intimately related to the level of the natural rate of output, itself an unobserved variable. Moreover, economic theory implies that the natural rate of interest varies over time in response to shifts to preferences and the growth rate of output. Household intertemporal utility maximization yields the relationship between the real rate and growth:

$$r = \frac{1}{\sigma} g_c + \theta, \quad (1)$$

where  $r$  is the real interest rate,  $\sigma$  denotes the intertemporal elasticity of substitution in consumption,  $g_c$  is the growth rate of per capita consumption, and  $\theta$  is the rate of time preference. Oliner and Sichel (2000) and Roberts (2001) provide evidence of shifts in the trend growth rate in the United States, suggesting one source of persistent movements in the natural rate of interest; changes in preferences and fiscal policy likely contribute as well to time variation in the natural rate. Based on the theoretical link between the natural rate of interest and the growth rate noted above, we assume that the law of motion for the natural rate of interest, denoted by  $r^*$ , is given by:

$$r_t^* = c g_t + z_t, \quad (2)$$

where  $g_t$  is the trend growth rate of the natural rate of output and  $z_t$  captures other determinants of  $r^*$ , such as households' rate of time preference.

Given that the key determinants of the natural rate of interest are unobserved, we apply the Kalman filter to estimate jointly the natural rates of interest, output, and trend growth.<sup>3</sup> Econometric identification of the natural rate of interest is achieved by specifying a simple reduced-form "IS" equation similar to that in Rudebusch and Svensson (1999), where the output gap (the percent deviation of real GDP from the natural rate of output) is determined by its own lags, a moving average of the lagged real funds rate gap (the difference between the ex ante real funds rate and  $r^*$ ), and a serially uncorrelated error:

$$\tilde{y}_t = a_{y,1} \tilde{y}_{t-1} + a_{y,2} \tilde{y}_{t-2} + \frac{a_r}{2} \sum_{j=1}^2 (r_{t-j} - r_{t-j}^*) + \epsilon_{1,t}, \quad (3)$$

where  $\tilde{y}_t = 100 * (y_t - y_t^*)$  denotes the output gap,  $y_t$  is the logarithm of real GDP,  $y_t^*$  is the logarithm of the unobserved natural rate of output, and  $r_t$  is the ex ante real federal funds

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<sup>3</sup>Our approach to estimating the level and growth rate of the natural rate of output builds on the work of Watson (1986), Clark (1987), and Kuttner (1994) with a key difference in that these authors do not include an interest rate term in their models.

rate.<sup>4</sup> In constructing the ex ante real rate, we proxy inflation expectations with the forecast of the four-quarter-ahead percent change in the price index for personal consumption expenditures excluding food and energy (“core PCE prices”) generated from a univariate AR(3) of inflation estimated over the prior 40 quarters. Note that the output gap is related to the real short-term rate; an alternative specification replaces the real funds rate gap with a real long-term bond rate gap, as in Fuhrer (1997). However, because of limited availability of data on long-term inflation expectations, implementation of such an approach necessitates further identifying assumptions regarding long-run expectations formation that take us away from the simple VAR framework, and we leave this to further research.

The core PCE price inflation rate,  $\pi_t$ , is assumed to be determined by its own lags, the lagged output gap, two variables measuring relative price shocks—core import (excluding petroleum, computers, and semiconductors) price inflation,  $\pi^I$ , and lagged crude imported oil price inflation,  $\pi^O$ —and a serially uncorrelated error:

$$\pi_t = B_\pi(L)\pi_{t-1} + b_y\tilde{y}_{t-1} + b_i(\pi_t^I - \pi_t) + b_o(\pi_{t-1}^O - \pi_{t-1}) + \epsilon_{2,t}. \quad (4)$$

We include eight lags of inflation in the equation and impose the restriction—not rejected in our sample—that the sum of the coefficients on lagged inflation equal unity. This specification is similar to that used in the literature to estimate the natural rate of unemployment (Gordon (1998), Brayton, Roberts and Williams (1999), Laubach (2001)). Equations (3) and (4) constitute the measurement equations of the basic version of our state-space model. We also consider an augmented model that includes an additional measurement equation that relates detrended private nonfarm employee hours,  $\tilde{h}_t$ , to the output gap, as in Roberts (2001):

$$\tilde{h}_t = f_1\tilde{y}_t + f_2\tilde{y}_{t-1} + f_3\tilde{h}_{t-1} + \epsilon_{6,t}, \quad (5)$$

where  $\tilde{h}_t$  denotes percent deviations of private nonfarm employee hours  $h_t$  from a log-linear trend  $h_t^*$ .<sup>5</sup> The purpose of including this equation is to add more information that may

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<sup>4</sup>For the nominal funds rate, we use the quarterly average of the annualized rate; because the federal funds rate frequently fell below the discount rate prior to 1965, we use the Federal Reserve Bank of New York’s discount rate prior to 1965.

<sup>5</sup>In principle,  $h_t^*$  could be specified as an I(2) process analogous to  $y_t^*$ . However, tests for an intercept shift in the raw hours data over our sample show no evidence for such a shift. Moreover, when attempting to construct a preliminary estimate of trend hours by estimating a trend-cycle decomposition based on a constant drift in trend hours, trend hours are estimated to be a simple time trend. These results differ from those of Roberts (2001) who normalizes hours by population and finds this series to be I(2).

improve the precision of the estimates of the output gap.

Turning to the transition equations of the model, we assume that the variable  $z$  defined in (2) follows an autoregressive process,

$$z_t = D_z(L) z_{t-1} + \epsilon_{3,t}. \quad (6)$$

We consider two specifications for  $z$ : (1)  $z$  is a stationary AR(2) process, and (2)  $z$  follows a random walk. We allow for shocks to both the level of the natural rate of output – the I(1) component – and its trend growth rate – the I(2) component. For reasons of parsimony, we specify a simple random walk model for both components:

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{4,t}, \quad (7)$$

$$g_t = g_{t-1} + \epsilon_{5,t}. \quad (8)$$

We assume  $\epsilon_{3,t}$ ,  $\epsilon_{4,t}$ , and  $\epsilon_{5,t}$  are serially uncorrelated and contemporaneously uncorrelated innovations. Equations (6)–(8) constitute the transition equations of our state-space model.

In applying the Kalman filter to this model, maximum likelihood estimates of the standard deviations of the innovations to  $z$  and the trend growth rate,  $\sigma_3$  and  $\sigma_5$ , are likely to be biased towards zero owing to the so-called “pile-up problem” discussed in Stock (1994). We therefore use the Stock and Watson (1998) median unbiased estimator to obtain estimates of the ratio  $\lambda_g \equiv \frac{\sigma_5}{\sigma_4}$ , and, in cases where  $z$  is nonstationary,  $\lambda_z \equiv \frac{\sigma_3}{\sigma_1} \frac{a_r}{\sqrt{2}}$ . We impose these ratios when estimating the remaining model parameters (including  $\sigma_1$  and  $\sigma_4$ ) by maximum likelihood. Because the pile-up problem does not arise for stationary unobserved processes, in the case where we assume that  $z$  is stationary, we estimate the standard deviation of its innovation,  $\sigma_3$ , by maximum likelihood simultaneously with the other model parameters.

Our estimation method proceeds in sequential steps. In the first step, we follow Kuttner (1994) and apply the Kalman filter to estimate the natural rate of output, omitting the real rate gap term from equation (3) and assuming that the trend growth rate,  $g$ , is constant. From this preliminary estimate of the natural rate of output we compute the median unbiased estimate of  $\lambda_g$ . If the unobserved component of the natural rate of interest,  $z$ , is assumed to be a random walk, we impose the estimated value of  $\lambda_g$  from the first step and include the real interest rate gap in the output gap equation under the assumption that  $z$  is constant. We estimate the five model equations and obtain an estimate of  $\lambda_z$ .

In the final step, we impose the estimated values of  $\lambda_g$  from the first step and  $\lambda_z$  from the second step (under the assumption that  $z$  follows a random walk; otherwise we estimate  $\sigma_3$

directly) and estimate the remaining model parameters by maximum likelihood as described by Harvey (1989).<sup>6</sup> We compute estimated confidence intervals and corresponding standard errors for the estimates of the states using Hamilton’s (1986) Monte Carlo procedure that accounts for both filter and parameter uncertainty.<sup>7</sup>

### 3 Estimation Results

All state-space models are estimated using U.S. data from 1961q1 to 2002q2. For the model consisting of the measurement equations (3)–(4), the Stock-Watson median unbiased estimates indicate that the trend growth rate and the  $z$  component of the natural rate of interest each exhibit relatively modest variation over time. The estimated value of  $\lambda_g$  is 0.042, with the 90 percent confidence interval, computed by Monte Carlo simulations, ranging from 0 to 0.110.<sup>8</sup> Assuming  $z$  follows a random walk and using the point estimate of  $\lambda_g$ , we obtain an estimate of  $\lambda_z$  of 0.058, with the 90 percent confidence interval covering 0.006 to 0.106.

Table 1 reports estimation results for the full set of model parameters for different model specifications and different assumed values of  $\lambda_g$  and  $\lambda_z$ . In all cases, the coefficient relating the output gap to the real rate gap,  $a_r$ , is negative and statistically significant. The first column reports results where we assume that  $z$  is a stationary AR(2) process. In this case, the coefficient relating the natural rate of interest to the trend growth rate is estimated to be unity and is significantly different from zero. The second column reports the baseline results when  $z$  is assumed to follow a random walk; the coefficient estimates are quite similar to those of the previous case. We note that in the cases where  $z$  is nonstationary, any inferences regarding the coefficient  $c$  based on the standard errors are invalid owing to the potential for spurious correlation (Granger and Newbold (1974)), and we do not report statistics for the estimates of the coefficient  $c$  in those cases.

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<sup>6</sup>Estimates of the conditional expectation and covariance matrix of the initial state are computed using the GLS method discussed in Harvey (1989, p. 122), modified as discussed in Laubach (2001, p. 222).

<sup>7</sup>Note that these estimated confidence intervals hold the imposed values of  $\lambda_g$  and  $\lambda_z$  fixed but take into account the uncertainty regarding  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_4$ ,  $\sigma_6$ , and, thus, implicitly  $\sigma_3$  and  $\sigma_5$ .

<sup>8</sup>To compute this confidence interval, we use parameter estimates based on the second-stage model with a time-varying trend growth and a constant natural rate of interest to construct 10,000 simulated series of  $\{y_t^*, g_t\}$ . For each pair of simulated series we compute the exponential Wald statistic for an intercept shift in the first difference of  $y_t^*$ .

The coefficient estimates are robust to variation in the assumed values of  $\lambda_g$  and  $\lambda_z$  and the inclusion of the hours equation. The third through sixth columns examine the sensitivity of the random walk specification to uncertainty regarding the values of  $\lambda_g$  and  $\lambda_z$ . For example, in the third column, labeled “Low  $\lambda_z$ ,” we replace the median unbiased estimate of  $\lambda_z$  with the value corresponding to the lower bound of the 90 percent confidence interval, as indicated in the second line of the table; the fourth column reports the results where  $\lambda_z$  is set equal to the upper bound of the 90 percent confidence interval. Note that when we modify the assumed value of  $\lambda_g$ , we reestimate  $\lambda_z$  according to the procedures outlined in the previous section. The coefficients related to output gap and inflation dynamics are relatively insensitive to the assumptions regarding the values of  $\lambda_z$  and  $\lambda_g$ , and, based on the log likelihood values, the data are unable to distinguish clearly between the first six alternative specifications. The final column reports results for the baseline random walk model augmented with the equation for hours. The coefficients of this model differ somewhat from those of the other models, but the strong relationship between the trend growth and the natural rate of interest remains.

Even with hindsight, the natural rate of interest is estimated imprecisely, with the sample average standard error of between 0.90 and 2.78 percentage points, as shown in the section of Table 1 labeled “S.E. (sample ave.)” Although the specification that includes the equation for employee hours, shown in the final column of the table, dramatically improves the precision of the estimate of the output gap, the natural rate of interest remains very imprecisely estimated. Overall, the imprecision of the natural rate estimates is larger the greater are the assumed values of  $\lambda_g$  and  $\lambda_z$ , that is, the greater the variation and uncertainty surrounding estimates of the trend growth rate and the unobserved process  $z$ . But, even in the cases where  $r^*$  is nearly constant, the standard error of nearly 1 percentage point is quite large and of similar magnitude to those associated with estimates of a time-varying natural rate of unemployment reported in Staiger et al. (1997).

The imprecision in estimates of the natural rate of interest is even greater when examining the one-sided estimates of  $r^*$  that correspond more closely to “real-time” estimates, in that only current and past observations are used in estimating the state. (Because the full sample is used in estimating the model parameters, the analogy, however, is not exact.) The final observation standard errors shown at the bottom of the table provide a measure of the imprecision of one-sided estimates, assuming the true values of  $\lambda_g$  and  $\lambda_z$  are known.

Figure 1 illustrates the differences between the one- and two-sided estimates for the baseline specification. The one-sided filter estimates of the natural rate of interest are shown by the dashed line, and the two-sided smoothed estimates are shown by the solid line. For comparison, the sample mean of the smoothed estimate – about 3 percent – is shown by the horizontal dashed line. The sample standard deviation of the difference between the one- and two-sided estimates is about 0.9 percentage point over the sample.

The standard errors reported above ignore uncertainty regarding  $\lambda_g$  and  $\lambda_z$  and model uncertainty and thus provide lower bounds on the “true” degree of uncertainty regarding these estimates. As seen in the top panel of Figure 2, the estimates of the natural rate of interest are sensitive to the assumptions regarding  $z$ . This figure shows the smoothed (two-sided) estimates of the unobserved processes implied by our model estimates for the different assumptions regarding the process for  $z$  corresponding to the first four columns of Table 1. In particular, the AR(2) specification for  $z$  yields estimates of  $r^*$  that are highly variable over time; in contrast, the random walk specification with the lower bound value of  $\lambda_z$  yields  $r^*$  estimates that are nearly constant. Estimates of the trend growth rate and the output gap, however, are not sensitive to the specification of  $z$ , as seen in the middle and bottom panels of the figure.

The estimates of the natural rate of interest are also sensitive to the assumptions regarding estimation of the natural rate of output and its trend growth rate. Figure 3 shows the sensitivity of the smoothed estimates of the natural rate of interest, the trend growth rate, and of the output gap to changes in the specification for the processes for the natural rate of output and its trend growth rate, corresponding to the final three columns of Table 1. The baseline estimates are also shown for comparison. Changes in the value of  $\lambda_g$  have a dramatic effect on the magnitude of time-variation in the trend growth rate, as seen in the middle panel. These different estimates of the trend growth rate do not, however, lead to significantly different estimates of the output gap, shown in the bottom panel. The estimates of the natural rate of interest are affected by the degree of variation in the trend growth rate owing to the relationship between  $r^*$  and the growth rate. Relative to the baseline specification, the model augmented with the hours equation yields higher estimates of the natural rate of output during most of the first half of sample and lower estimates during much of the second half. As a result, the estimates of the natural rate of interest from the augmented model are lower than those from the baseline model

during the earlier period and generally higher during the latter period. Taken together, these results cast considerable doubt on the ability to estimate the natural rate of interest with much precision in real time.

Given the imprecision of the Kalman filter natural rate estimates, we now briefly examine the properties of other time series methods. Specifically, we compare our Kalman filter estimates of the natural rate of interest to estimates based on two popular univariate filters of the ex post real rate, defined to be the difference between the funds rate and the realized core PCE inflation rate. The first univariate method is the Hodrick-Prescott (1997) filter, for which we use a value of the smoothing parameter  $\lambda$  of 6400. The second method is the Band-Pass filter described by Baxter and King (1999), for which we use a 60-quarter window as in Staiger, Stock and Watson (2002).<sup>9</sup> Figure 4 shows the baseline two-sided Kalman filter estimates and the two-sided estimates from the univariate filters. As seen in the figure, the estimates from the two univariate filter methods are very similar.

The univariate filter estimates arguably provide a misleading picture of the natural rate of interest because they ignore information from movements in inflation. By their nature as two-sided weighted averages, the univariate estimates imply that real rates were on average close to the natural rate during the 1960s and early 1970s, even though inflation rose steadily during that period. Similarly, the univariate filters inappropriately ascribe a large portion of the disinflationary policy action during the late 1970s and early 1980s to an upward shift in the natural rate. In contrast, the Kalman filter estimates show that policy was on average stimulative during the 1960s and early 1970s and that the natural rate moved very little during the later disinflationary period. For these reasons, the Kalman filter approach appears to do better at estimating lower frequency movements in the natural rate of interest than univariate time-series methods. The same conclusion holds true for the one-sided Kalman and univariate filter estimates. Indeed, the latter are very sensitive to the actual data and thus especially poorly suited to estimating trends in real time, as discussed in Orphanides and van Norden (2002) and Orphanides and Williams (2002).

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<sup>9</sup>We chose these parameterizations of the filters so that the resulting estimates correspond to the lower frequency component similar to that from the Kalman filter. Univariate filter estimates that impose less smoothing are qualitatively similar to those reported here, but are more sensitive to the actual data.

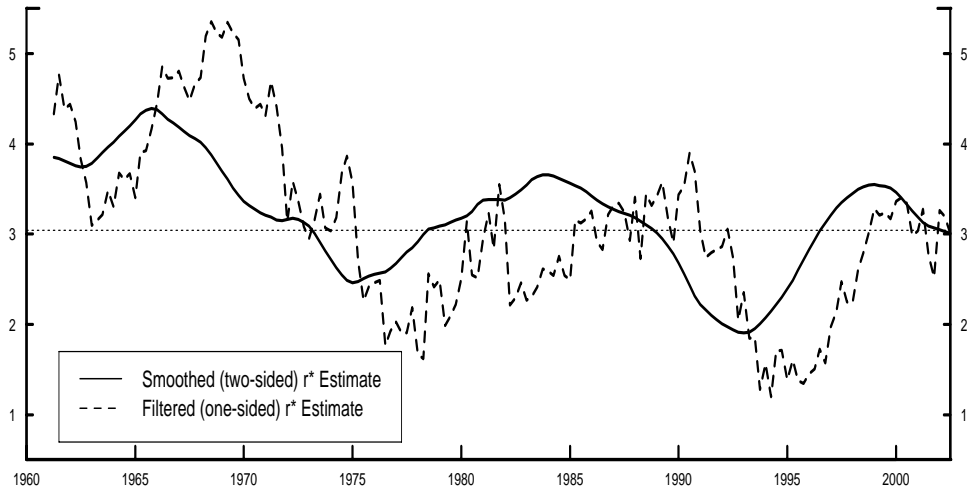
## 4 Conclusion

In this paper, we jointly estimated the natural rates of interest and output and the trend growth rate using the Kalman filter. The natural rate of interest shows significant variation over the past forty years in the United States, with variation in the trend growth rate an important determinant of changes in the natural rate, as predicted by theory. These results are robust to changes in specification. However, estimates of a time-varying natural rate of interest, like those of the natural rates of unemployment and output, are very imprecise and are subject to considerable real-time mismeasurement. These results suggest that this source of uncertainty needs to be taken account of in analyzing monetary policies that feature responses to the natural rate of interest.

Table 1: Parameter Estimates

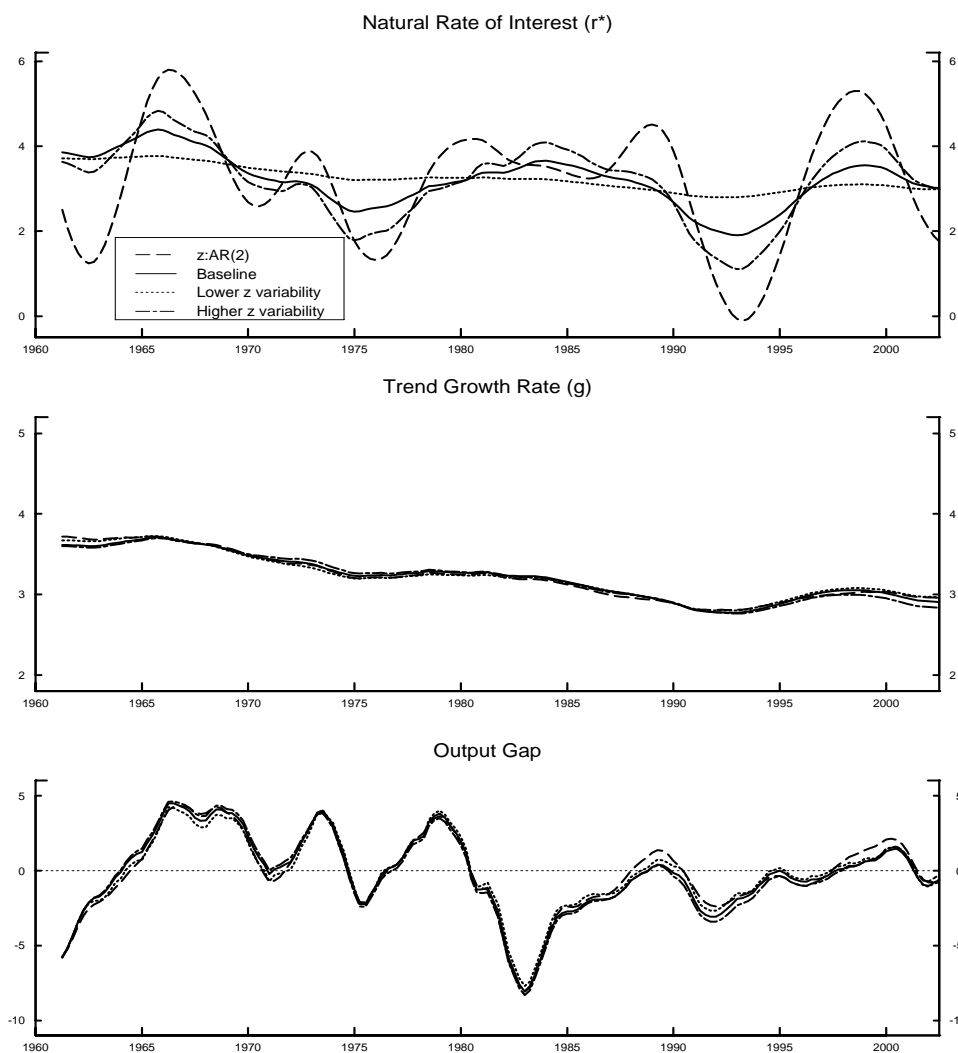
Parameter	Stationary	Random Walk Model for $z$					
	$z \sim \text{AR}(2)$	Baseline	Low $\lambda_z$	High $\lambda_z$	Low $\lambda_g$	High $\lambda_g$	Hours
$\lambda_g$	.042	.042	.042	.042	.000	.110	.043
$\lambda_z$	.042 (.26)	.058	.006	.106	.012	.047	.022
$\sum a_y$	.925	.945	.945	.945	.946	.948	.929
$a_r$	-.119 (3.45)	-.098 (3.81)	-.088 (3.79)	-.109 (3.78)	-.083 (3.77)	-.122 (3.71)	-.063 (2.82)
$b_y$	.057 (1.89)	.043 (1.63)	.060 (1.89)	.035 (1.53)	.058 (1.92)	.032 (1.32)	.089 (2.75)
$c$	1.008 (4.09)	1.068	1.011	1.010	–	1.043	.853
$\sigma_1 (\tilde{y})$	.256	.387	.435	.348	.407	.282	.496
$\sigma_2 (\pi)$	.727	.731	.727	.733	.728	.735	.724
$\sigma_3 (z)$	.126	.323	.042	.480	.084	.153	.244
$\sigma_4 (y^*)$	.644	.605	.580	.619	.608	.642	.546
$\sigma_5 (g)$	.108	.102	.097	.104	–	.282	.094
$\sigma_{r^*} = \sqrt{c^2\sigma_5^2 + \sigma_3^2}$	.167	.340	.107	.491	.084	.332	.256
Log Likelihood	-373.5	-376.7	-375.6	-377.0	-375.8	-377.5	-485.0 <sup>1</sup>
S.E. (sample ave.)							
$r^*$	2.78	1.88	.90	2.30	.98	1.51	2.66
$g$	.43	.48	.40	.53	.23	.72	.44
$y^*$	2.20	3.02	1.87	3.87	1.92	3.25	.56
S.E. (final obs.)							
$r^*$	4.22	2.61	.97	3.38	1.13	1.96	3.83
$g$	.59	.63	.52	.71	.23	.90	.58
$y^*$	3.01	4.23	2.02	5.88	2.17	4.23	.58
Notes: $t$ statistics are indicated in parentheses; $\sigma_5$ is expressed at annual rate.							
1. The log likelihood for the model augmented with equation for employee hours is not comparable to the others.							

Figure 1: One- vs. Two-sided Estimates of the Natural Rate of Interest (Baseline Model)



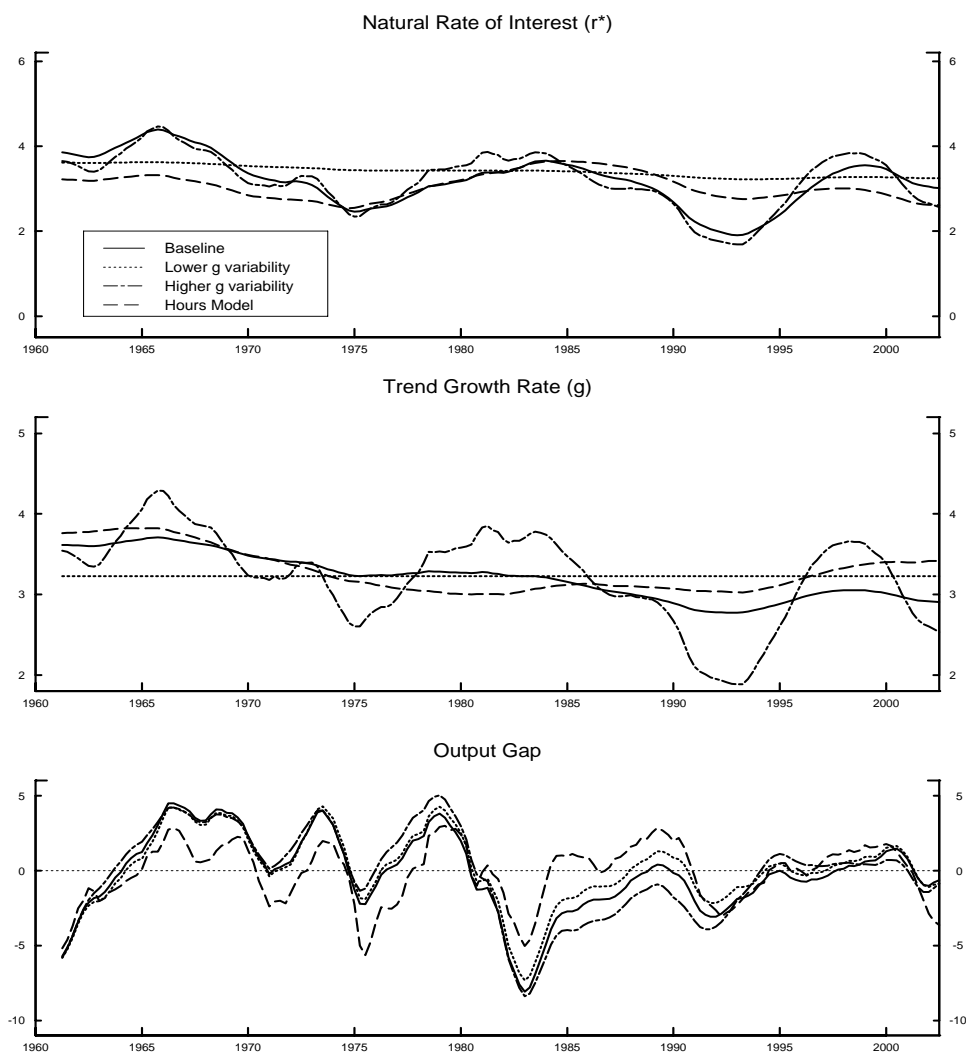
Notes: The solid line shows the smoothed (two-sided) estimates of the natural rate of interest for the baseline specification in which  $z$  is assumed to follow a random walk. The dashed line shows the corresponding filter (one-sided) estimates.

Figure 2: Sensitivity of Smoothed Estimates to Assumptions Regarding  $z$



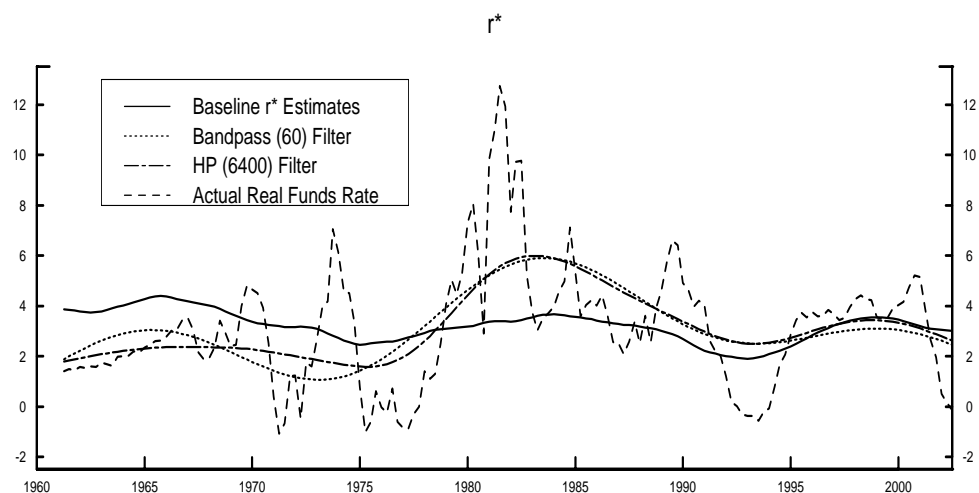
Notes: The dashed lines show the smoothed (two-sided) estimates of the unobserved states for the model in which  $z$  is assumed to follow an AR(2) process. The solid lines show the corresponding estimates for the baseline case where  $z$  is assumed to follow a random walk. The dotted lines show the estimates for the lower bound value of  $\lambda_z$ ; the dash-dot lines show the values for the upper bound value of  $\lambda_z$ .

Figure 3: Sensitivity of Estimates to Assumptions Regarding Natural Rate of Output



Notes: The solid lines show the smoothed (two-sided) estimates of the unobserved states for the baseline case where  $z$  is assumed to follow a random walk. The dotted lines show the estimates for the lower bound value of  $\lambda_z$ ; the dash-dot lines show the values for the upper bound value of  $\lambda_z$ . The dashed line shows the estimates for the baseline model augmented with the hours equation.

Figure 4: Univariate Estimates of the Natural Rate of Interest



Notes: The solid line shows the smoothed (two-sided) Kalman filter estimates of the natural rate of interest for the baseline case where  $z$  is assumed to follow a random walk. The dotted line shows the Bandpass filter estimates using a 60 quarter window. The dash-dot line shows the HP filter estimates using a smoothing parameter of 6400. The dashed line shows the actual ex post real funds rate.

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